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Differential Amplifiers

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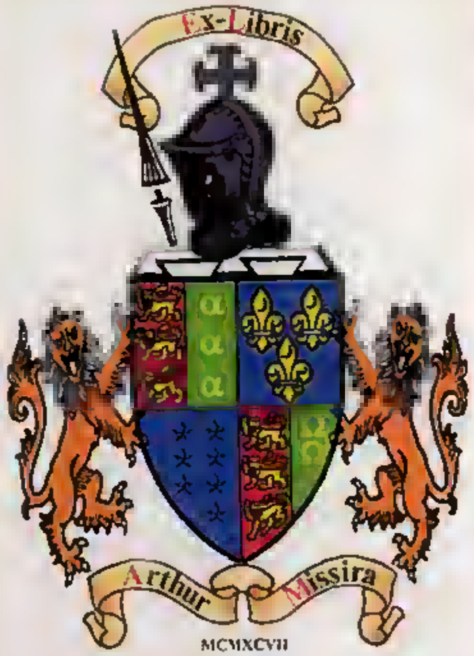
R. D. MIDDLEBROOK



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Differential Amplifiers

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R. D. Middlebrook, SERIES EDITOR

Differential Amplifiers
by R. D. Middlebrook

Differential Amplifiers

Their Analysis and Their Applications in Transistor d-c Amplifiers

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Foreword

There are many books on the subject of electronic circuits. Most of them are texts; they emphasize the broad principles and analysis of many classes of circuits in a manner suited to a first- or second-year college course. Armed with these fundamentals, the electronic engineer in his first job finds himself faced with the task of designing a circuit to perform a required function—and soon discovers that the information in his favorite texts is not enough. He must resort to searching through an immense backlog of technical journals to learn how the fundamental circuits are augmented and modified for practical applications.

Books which expound the principles of circuits, and methods for their analysis, are, of course, essential prerequisites to an engineering education. At the same time there is a great need for more books, useful to the graduate engineer, that assume the fundamentals and carry the design of electronic circuits to their practical realization in instruments and equipment to perform required functions. The vast engineering development work described in the technical journals, scattered in both time and space, must be distilled; comparisons between various modifications evaluated; relative merits of one configuration over another investigated; and, most importantly, the inevitable engineering compromises recognized and discussed.

The Wiley Monograph Series on Electronic Circuits is intended to fill the gap between circuit principles and practical design in certain selected areas. Since the requirements of the various areas differ, it is neither possible nor desirable to lay down a specific approach or format for each monograph in the series. However, I hope that each monograph will serve the practicing design engineer as a survey, a text, and a manual for a particular class of circuits.

R. D. Middlebrook
Series Editor

Division of Engineering and Applied Science
California Institute of Technology
December, 1962

Preface

Differential amplifiers, also known as difference amplifiers, cathode- or emitter-coupled amplifiers, and push-pull amplifiers, are frequently employed in several branches of instrumentation. Differential amplifiers possess in general a three-terminal input and a three-terminal output, and it is usual to describe their performance with respect to common-mode (CM) and differential-mode (DM) signals. Much of the early work on differential amplifiers was for application to biological instrumentation, in which it was necessary to amplify very small DM signals to the exclusion of large interference CM signals. These instruments used vacuum tubes exclusively. Later, the availability of improved transistors led to the development of high-performance solid-state d-c differential amplifiers for use in telemetry and other signal-conditioning systems. Because of the almost complete isolation of the histories of vacuum-tube and transistor differential amplifiers, there is a need to integrate the theories of the two classes and to apply the principles learned over the years with tube amplifiers to modern high-performance solid-state circuits.

The three-terminal nature of both input and output of differential amplifiers requires the definition of more parameters to describe their performance than are necessary for the more familiar type of amplifier in which one input and one output terminal are grounded. Thus one defines, among other quantities, a DM gain, a CM gain, and DM and CM input impedances. Since the primary purpose of a differential amplifier is to amplify DM but not CM signals, it was early recognized that the ratio of the DM gain to the CM gain, called the discrimination factor, should be large. However, it became apparent that there are also other important effects, which arise from interaction between the DM and CM signals.

These effects are due to magnitude unbalances in the circuit symmetry and are described by additional performance parameters, such as the DM-to-CM and CM-to-DM transfer gains, the CM and DM rejection factors, and the power supply rejection factors.

One objective of this monograph is to clarify and define the important performance parameters of a differential amplifier. A differential amplifier is an example of a symmetrical circuit, and if the circuit is balanced the bisection theorem may be invoked to permit separate analysis for CM and for DM signals. If the circuit is unbalanced, however, the analysis becomes in general much more complicated. Another objective of this monograph is to develop an analysis technique whereby the effects of small percentage unbalances in the circuit symmetry may be incorporated through an extension of the bisection theorem. The method is called "sequential" since the analysis is broken down into a series of individually simple steps and thus affords reasonable algebraic simplicity in the solution of even quite complex circuits.

I hope that the material will be of broad interest to those concerned with general electronic-circuit design and with general circuit analysis techniques. The material will also be of specific interest to those directly concerned with the analysis and design of differential amplifiers, since the philosophy and the methods treated in detail are applicable to a wide variety of practical amplifiers.

The properties of differential amplifiers and the analysis technique for unbalanced symmetrical circuits are illustrated by application to two specific transistor circuits. The first is a basic single-stage d-c differential amplifier, and the second is a more sophisticated two-stage d-c differential amplifier with common-mode negative feedback. These circuits are chosen not only because they are convenient vehicles for the discussion, but also because they are important in their own right for practical applications. Typical numerical values are employed for the circuit components in order to illustrate the considerable effects that even small unbalances can have on the circuit performance parameters and to facilitate comparison between the two particular amplifier circuits chosen for discussion. Since these effects are numerous and complex, attention is given throughout to the problems of ensuring that the algebra remains under control and of expressing the equations in physically interpretable forms. Although at first glance some of the equations may appear formidable, they are merely long rather than complex, and the terms are easily identified.

Although only two specific amplifier circuits are treated in detail, the sequential method is applicable in general to the analysis of symmetrical circuits of any kind, and it permits the treatment of small unbalances in

vacuum-tube or transistor d-c or a-c differential amplifiers to be conducted on a straightforward yet comprehensive basis.

It is impossible, as always, to acknowledge indebtedness to all those who contribute to a book in real but unseen ways. Special thanks are, however, due to Microdot Inc., South Pasadena, California, who provided me the opportunity to work with direct-coupled d-c amplifiers. My wife, Frances, who gave me constant encouragement through many nights and weekends of work, is responsible for the completion of the monograph in so short a time. My secretary, Cynthia Markiewicz, who battled successfully with some of the worst equations she has ever seen, contributed tangibly to the accuracy of the manuscript, and any remaining errors are entirely my own responsibility.

R. D. Middlebrook

Division of Engineering and Applied Science
California Institute of Technology
Pasadena, California
December, 1962

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Differential Amplifiers

Chapter One

The nature of differential amplifiers

Differential amplifiers constitute a special class of circuits whose function is to amplify the difference between two signals, regardless of their individual values. Considerable impetus was given to the development of vacuum-tube differential amplifiers by their suitability for biological measurements, and more recently transistor differential amplifiers have assumed importance in telemetry applications. The purpose of this monograph is threefold: First, to clarify and define the properties of differential amplifiers in general; second, to develop a method for their analysis, in order to provide a useful foundation for design; and third, to illustrate their properties and the analysis method through consideration of some specific, and practical, transistor d-c differential-amplifier circuits.

The three objectives are to some extent approached concurrently, in that the specific examples are used both to establish the basic properties of differential amplifiers and to introduce and illustrate the analysis method. Nevertheless, the analysis techniques and the performance properties are applicable not only to differential-amplifier circuits in particular, but also to the more general class of *symmetrical circuits*, so called because they can be topologically divided into two mirror-image parts. A bridge network is an example of a passive symmetrical circuit; active symmetrical circuits include not only differential amplifiers as such, but also push-pull amplifiers and cathode- or emitter-coupled amplifiers which constitute parts of unsymmetrical circuits.

The need for differential amplifiers is established in Section 1.1 through consideration of two typical instrumentation systems. In Sections 1.2 and 1.3 the historical development of differential-amplifier theory is traced with a simple vacuum-tube example. The various properties of a differential

amplifier are introduced, with emphasis on those related to circuit unbalances. Finally, in Section 1.4, the analysis problem of differential amplifiers is summarized, and the steps to be followed for its solution in the later chapters are outlined.

1.1 The Need for Differential Amplifiers

An amplifier possesses in general three sets of terminals: An input terminal pair, an output terminal pair, and a terminal pair at which operating power is supplied. In most amplifiers one terminal of each pair is connected to a common point, usually considered to be at "zero" or "ground" potential. Such an amplifier is said to have a "single-ended input" and a "single-ended output." In certain applications, however, an amplifier with single-ended input is not satisfactory. It is necessary instead that neither input terminal be grounded, in which case the amplifier is said to have a "floating input." An amplifier that provides an output proportional to the difference between the signals at the two terminals of an input floating with respect to ground, while preventing any output arising from a signal which is common to the two input terminals, is called a "difference" or "differential" amplifier. The need for differential amplifiers may be understood from the following two examples.

Electroencephalography is the study and interpretation of electrical potentials that occur on the surface of the brain. In the usual technique[1] several electrodes are placed at various points on the scalp, and the potential differences between pairs of electrodes are amplified and recorded for examination. A given pair of electrodes constitutes a signal source, and may be represented by a voltage generator and a series resistance. The voltage is the quantity of interest, and the series resistance is the sum of the physiological resistance between the electrodes, the contact resistance of each electrode, and the electric-circuit resistance. A functional diagram of an instrumentation system using a single-ended input amplifier is shown* in Fig. 1.

Unfortunately, completely spurious results are likely to be obtained at the amplifier output because of interference introduced within the signal source. The most serious source of interference is 60-cps potentials, which are inevitably present in typical surroundings, and which are coupled into the amplifier input circuit through the fairly high, and variable, source

* For convenience in reference, all the figures and tables are collected at the end of the text.

resistance. Interference currents flowing round the loop indicated in Fig. 1 can develop input voltages to the amplifier that completely swamp the few tens of microvolts which constitute the desired signal.

To alleviate the interference problem it is necessary to employ an amplifier with a floating input, as shown in Fig. 2. If the impedance of each input terminal to ground is sufficiently high, the interference voltage is not able to inject a significant signal into the input loop. It may be noted that the amplifier, in addition to having high impedance at both input terminals, must also be capable of ignoring the interference voltage which appears at both input terminals simultaneously. Much of the early work on differential amplifiers was stimulated by the requirements of instrumentation for electroencephalography.

A second example of the need for differential amplifiers may be found in the field of physical measurements. It is often important to know the mechanical strain to which a metal structure is subjected, and one way to determine this is to attach a strain-gauge bridge to the metal surface.[2] Measurement of the strain in a rocket engine casing is a typical use, and in this case the associated amplifier will be in a blockhouse some distance away. A strain-gauge bridge consists of four resistances in a bridge configuration, and one or perhaps two of the resistances are strain sensitive. The bridge is activated by a d-c supply voltage, and the voltage developed between the other pair of bridge terminals is a function of the strain to be measured. Since one terminal of the bridge supply voltage is grounded, it follows that neither terminal at which the bridge output voltage appears may be grounded; otherwise one of the bridge arms would be shorted out. It is therefore necessary that the amplifier to which the bridge is connected have floating input terminals. A functional diagram of the system is shown in Fig. 3.

Interference voltages are also a serious problem in strain-gauge instrumentation. Because of the distance between the bridge and the amplifier, the bridge "ground" is rarely at the same potential as the amplifier ground, and an interference voltage due to "ground loop" currents may be represented as in Fig. 3. The interference signal may be of several volts magnitude and may contain large 400-cps components, since the frequency of the prime power in such installations is often 400 cps. It is again clear that the differential amplifier must have high impedance to ground from both input terminals, must also be capable of suppressing large voltages common to both input terminals, and at the same time must amplify the few millivolts between the input terminals that result from the desired bridge signal.

We consider next some basic amplifier circuits that are suitable for applications of the variety we have described.

1.2 Properties of the Basic Differential Amplifier as a Balanced Symmetrical Circuit

We have seen that a basic requirement of differential amplifiers is an input floating with respect to amplifier ground. At first sight it seems as though an adequate solution would be to provide a transformer-coupled input to an ordinary single-ended amplifier stage, such as a simple grounded-cathode vacuum-tube circuit. However, this solution is generally not satisfactory, because the input signals to be amplified often include d-c voltages, or at least frequencies so low that transformer coupling becomes impractical. It is moreover difficult to maintain sufficiently high input impedances with a transformer.

An alternative solution is to amplify individually the signal at each input terminal with respect to ground. The basic circuit is shown in Fig. 4 (bias arrangements are omitted). This amplifier is of the floating-input, floating-output type, and hence there are two classes of input and output signals: Common-mode (inphase), and differential-mode (antiphase). The common-mode (CM) signal is defined as the average of the voltages of the two terminals with respect to ground, or half their sum; the differential-mode (DM) signal is defined as half their difference. In the circuit of Fig. 4, as long as the two halves are exactly similar, the DM output is related only to the DM input and is independent of any CM input. Similar remarks apply to the CM output. We may hence define a CM gain and a DM gain; for this circuit they are equal.

The simple circuit of Fig. 4 is in principle suitable for amplifying signals from sources like those previously described. It should be noted, however, that the floating output must be used, and that the two halves of the circuit must be exactly balanced. In addition, the magnitude of the CM signal must be small enough to avoid overloading and consequent distortion of the CM signal. This effect can be particularly troublesome if several stages of the type in Fig. 4 are used in cascade, and it is aggravated by the frequent occurrence of CM signals at the source which may be several orders of magnitude larger than the desired DM signal.

In order to alleviate overloading problems it is desirable to reduce the CM gain of the amplifier without affecting the DM gain. One of the earliest modifications to the basic circuit was suggested by Offner,[3] who described a method of applying CM negative feedback to the input grids. An even simpler circuit modification, which leads to the same result, is to provide a common resistance between the two cathodes and ground, as shown in Fig. 5. This "cathode-coupled" amplifier is a very versatile circuit and is worth examining in some detail.

The qualitative performance of the circuit of Fig. 5 may best be understood by considering the separate application of DM and CM input signals. We assume for the present that the two tubes are identical and that the load resistances are equal. If a DM input signal is applied, input terminal *a* goes "up," and terminal *b* goes "down" by an equal amount. With the assumption of linear tube operation, the plate currents of the tubes respectively increase and decrease from their equal quiescent values, and these changes precisely cancel out in the cathode coupling resistance. The output voltages go "down" and "up" by equal amounts, and the DM gain of the amplifier is the same as the gain of each individual tube because the cathodes are at virtual ground. If a CM input signal is applied, both input terminals go "up" together, and both plate currents increase by equal amounts. These changes are additive in the cathode coupling resistance, and a negative feedback voltage is developed across it. The CM gain of the amplifier is therefore equal to that of each individual tube in the presence of cathode degeneration corresponding to a resistance twice the cathode coupling resistance. The factor 2 occurs because both plate current increments flow through this resistance. It is seen that the desired effect of making the DM-gain-to-CM-gain ratio greater than unity has been achieved.

The inherent discrimination of the circuit of Fig. 5 against common-mode in favor of differential-mode input signals leads to the name "differential amplifier." It is the basic circuit configuration, with numerous modifications, to be considered in the later chapters. It is appropriate to begin our analytical work by verifying the qualitative description of the circuit performance we have given.

The incremental equivalent circuit of the differential amplifier of Fig. 5 is shown in Fig. 6. Exact similarity of the two halves is retained, and for simplicity only a-c quantities are considered. Each tube is represented by the conventional low-frequency equivalent circuit containing the amplification factor μ and the plate resistance r_p. The two input voltages are v_a and v_b, and the two output voltages are v_{1a} and v_{1b}, all defined with respect to a-c ground. We are interested in how the CM and DM output voltages, $(v_{1a} + v_{1b})/2$ and $(v_{1a} - v_{1b})/2$ respectively, and the individual output voltages v_{1a} and v_{1b} depend on the input voltages v_a and v_b. Straightforward analysis of the circuit of Fig. 6 leads to the following results:

$$\left(\frac{v_{1a} + v_{1b}}{2}\right) = -\frac{\mu R_L}{R_L + r_p + 2R(1 + \mu)}\left(\frac{v_a + v_b}{2}\right) \tag{1.1}$$

$$\left(\frac{v_{1a} - v_{1b}}{2}\right) = -\frac{\mu R_L}{R_L + r_p}\left(\frac{v_a - v_b}{2}\right) \tag{1.2}$$

The results are expressed in terms of the CM and DM input voltages, rather than in terms of their individual values, to emphasize the condition that the CM operation can be considered quite independently of the DM operation.

The CM gain A_{cc} and the DM gain A_{dd} may be identified from Eqs. 1.1 and 1.2 as

$$A_{cc} \equiv \frac{\mu R_L}{R_L + r_p + 2R(1+\mu)} \tag{1.3}$$

$$A_{dd} \equiv \frac{\mu R_L}{R_L + r_p} \tag{1.4}$$

These expressions verify the qualitative conclusions that the DM gain is that of one tube alone, whereas the CM gain is that of one tube degenerated by twice the cathode coupling resistance.

By direct addition and subtraction of Eqs. 1.1 and 1.2 expressions for the individual output voltages may be obtained:

$$v_{1a} = -A_{cc}\left(\frac{v_a + v_b}{2}\right) - A_{dd}\left(\frac{v_a - v_b}{2}\right) \tag{1.5}$$

$$v_{1b} = -A_{cc}\left(\frac{v_a + v_b}{2}\right) + A_{dd}\left(\frac{v_a - v_b}{2}\right) \tag{1.6}$$

One important inference from these results is that *both* output voltages v_{1a} and v_{1b} are present even if *only one* input voltage v_a or v_b is applied. In particular, if $v_b = 0$,

$$v_{1a} = -\left(\frac{A_{dd} + A_{cc}}{2}\right)v_a \tag{1.7}$$

$$v_{1b} = +\left(\frac{A_{dd} - A_{cc}}{2}\right)v_a \tag{1.8}$$

If the CM gain A_{cc} is much less than the DM gain A_{dd}, as can be achieved by making R large, the two output voltages v_{1a} and v_{1b} tend to be equal in magnitude and opposite in phase in response to the input voltage v_a. This property of the circuit leads to its employment as a "cathode-coupled phase inverter," whose function is to provide a floating antiphase output from a single-ended input. A common use of a phase inverter circuit is as a driver for a push-pull audio power amplifier, and it may be noted that the basic differential amplifier of Fig. 5 may also be used as a push-pull stage. Usually, of course, the separate load resistances are replaced by a single load

resistance coupled to both anodes through a transformer with center-tapped primary. One function of the transformer in this application is to convert the floating output of the amplifier into a single-ended output, and its use is possible since frequency response is not required down to d-c.

In the more general application of the differential amplifier, in which both CM and DM input voltages are present, we have seen, by Eq. 1.2, that the DM output is a function only of the DM input. In many cases it is possible to utilize the floating output directly, as in electroencephalographic work where the ultimate electrical load is the drive coil of a pen recorder. In many applications, however, it is necessary to have available a single-ended output, as in telemetry work where the amplifier output is to be connected to a single-ended input device such as a multiplexer or a subcarrier oscillator. The basic differential amplifier of Fig. 5 is capable of supplying a single-ended output, if the output is considered to be between one output terminal and ground. It is nevertheless seen from Eq. 1.5 that the corresponding single-ended output voltage v_{1a} (or v_{1b}) contains components due both to the CM and to the DM input signals. To minimize the undesired CM component it is obviously necessary to employ a large DM-to-CM gain ratio A_{dd}/A_{cc}. This reason for requiring a large gain ratio is in addition to that, previously mentioned, of avoiding overloading effects.

The ratio of the DM gain to the CM gain, A_{dd}/A_{cc}, is an important parameter of a differential amplifier and is called the "discrimination factor." The desired large value of the discrimination factor can be achieved by using a large value of cathode coupling resistance R. There are practical limitations on the magnitude of R, however, because of the quiescent d-c voltage drop across it; the larger the value of R, the larger the negative supply voltage at the bottom end of R needed to maintain the cathodes at near-ground potential. A very elegant way of obtaining a high effective (dynamic) value of R, while retaining a comparatively low d-c voltage drop, has been suggested by Goldberg.[4] The modification consists of replacing the resistance R by a "constant-current" pentode tube, preferably with cathode degeneration from a resistance R_k, as shown in Fig. 7. The inherently high plate resistance r_p' of the pentode is increased to an effectively even higher value closely equal to $r_p'(1 + g_m'R_k)$, where g_m' is the transconductance of the pentode.

Before proceeding further, let us consider a numerical example, in order to obtain an idea of the magnitudes involved in a typical circuit. Suppose that the parameters in the differential-amplifier circuit of Figs. 5 and 6 are as follows:

$$\mu = 20 \qquad R_L = 10\text{ k}$$
$$r_p = 10\text{ k} \qquad R = 10\text{ k}$$

The CM and DM gains, from Eqs. 1.3 and 1.4, are

$$A_{cc} = 0.455 \tag{1.9}$$

$$A_{dd} = 10 \tag{1.10}$$

Note that because of the large CM negative feedback produced by the cathode coupling resistance R the CM gain is less than unity. The discrimination factor is $A_{dd}/A_{cc} = 22$. If the DM input signal $(v_a - v_b)/2$ is 10 mv, as might be obtained from a strain-gauge bridge in the configuration of Fig. 3, the component of the single-ended output voltage v_{1a} produced by this signal is 10×10 mv $= 0.1$ v, from Eq. 1.5. However, this output voltage could equally well have been produced by a CM input voltage $(v_a + v_b)/2$ equal to 0.1 v$/0.455 = 0.22$ v. A CM input voltage of this magnitude, or even greater, could easily occur in a practical system, and hence observation of the single-ended output voltage cannot tell us whether the input signal was a certain DM voltage, or a CM voltage 22 times as great, or some combination of the two. The amplifier performance in accepting DM and discriminating against CM inputs is obviously not very good.

The differential-amplifier performance derived in this numerical example can be considerably improved by replacement of the cathode coupling resistance R by a constant-current pentode, as in the modified circuit of Fig. 7. If the additional circuit has parameters

$$g_m' = 2000\ \mu\text{mho} \qquad R_k = 1\ \text{k} \qquad r_p' = 1\ \text{M}\Omega$$

the effective value of R to be used in the gain expressions of Eqs. 1.3 and 1.4 is $r_p'(1 + g_m'R_k) = 3$ MΩ. The resulting values of the two gains are

$$A_{cc} = 0.00159 \tag{1.11}$$

$$A_{dd} = 10 \tag{1.12}$$

The DM gain A_{dd} is of course unaltered, but the CM gain A_{cc} is very much smaller than in the previous numerical example. As a result, the discrimination factor is increased to $10/0.00159 = 6300$, and a CM input voltage to the differential amplifier would have to be 6300 times as large as a DM input to give the same value of single-ended output voltage. The discrimination against CM input voltages is obviously vastly improved over that in the previous performance.

In this section we have discussed some of the properties of the basic differential amplifier, and have shown how a common-cathode coupling resistance of large effective value can improve the performance. We consider next some very important additional considerations in the performance of the basic differential amplifier.

1.3 The Important Effects of Circuit Unbalances

It would appear from the discussion of the previous section that a differential amplifier of the type shown in Fig. 5 or Fig. 7 is capable of providing high discrimination against CM input signals. The equations describing the performance indicate that the only requirement for achieving arbitrarily high discrimination is as large a value as possible of the coupling resistance R, or of its effective value if more sophisticated circuits are used.

Unfortunately, the problem is not as simple as this. It has been assumed so far that the two halves of the circuit are identical; however, inevitable unbalances in the symmetry of a differential amplifier can permit an undesired DM output to be developed from a CM input. We may investigate the nature of this additional circuit property by reconsidering the basic differential amplifier of Fig. 5 and its equivalent circuit in Fig. 6.

As the simplest illustration of the effect of unbalances in the two halves of the circuit, let us suppose that all corresponding elements remain balanced except the tube amplification factors. Let the modified amplification factors be $\mu + \delta\mu$ and $\mu - \delta\mu$ for the left- and right-hand tubes, respectively. As before, we are interested in how the two output voltages, and their CM and DM components, depend on the CM and DM input voltages.

From the equivalent circuit of Fig. 6, with incorporation of the unbalanced μ's, considerably lengthier algebra than that used before leads to the following equations:

$$\left(\frac{v_{1a} + v_{1b}}{2}\right) = -\frac{\mu R_L}{r_p + R_L + 2R(1+\mu)} \left[\left(\frac{v_a + v_b}{2}\right) + \frac{\delta\mu}{\mu}\left(\frac{v_a - v_b}{2}\right)\right] \tag{1.13}$$

$$\left(\frac{v_{1a} - v_{1b}}{2}\right) = -\frac{\mu R_L}{r_p + R_L}\left[\left(1 - \frac{(\delta\mu/\mu)^2 2R\mu}{r_p + R_L + 2R(1+\mu)}\right)\left(\frac{v_a - v_b}{2}\right) + \left(\frac{\delta\mu}{\mu}\right)\left(1 - \frac{2R\mu}{r_p + R_L + 2R(1+\mu)}\right)\left(\frac{v_a + v_b}{2}\right)\right] \tag{1.14}$$

The most significant feature of these results, as compared with Eqs. 1.1 and 1.2, is the presence in each expression of terms in both the DM and the CM input voltages. These "cross-coupling" terms disappear when the circuit elements are balanced in value, that is, when $\delta\mu = 0$ in the present

example. It is convenient to express the cross-coupling properties by means of two transfer gains, so that now four gain parameters may be identified from Eqs. 1.13 and 1.14 as

$$A_{cc} \equiv \frac{\mu R_L}{r_p + R_L + 2R(1+\mu)} \tag{1.15}$$

$$A_{dd} \equiv \frac{\mu R_L}{r_p + R_L}\left(1 - \frac{(\delta\mu/\mu)^2 2R\mu}{r_p + R_L + 2R(1+\mu)}\right) \tag{1.16}$$

$$A_{cd} \equiv \frac{\delta\mu}{\mu}\frac{\mu R_L}{r_p + R_L} \tag{1.17}$$

$$A_{dc} \equiv \frac{\delta\mu}{\mu}\frac{\mu R_L}{r_p + R_L}\left(1 - \frac{2R\mu}{r_p + R_L + 2R(1+\mu)}\right) \tag{1.18}$$

The CM gain A_{cc} is a first-order quantity and is unaffected by unbalanced μ's, and the DM gain, also a first-order quantity, differs from its original value by only a third-order term in $(\delta\mu/\mu)^2$. In contrast, the two transfer gains A_{cd} and A_{dc} are second-order quantities, and have direct dependence on the fractional unbalance $\delta\mu/\mu$. With these definitions, Eqs. 1.13 and 1.14 may be written

$$\left(\frac{v_{1a} + v_{1b}}{2}\right) = -A_{cc}\left(\frac{v_a + v_b}{2}\right) - A_{cd}\left(\frac{v_a - v_b}{2}\right) \tag{1.19}$$

$$\left(\frac{v_{1a} - v_{1b}}{2}\right) = -A_{dd}\left(\frac{v_a - v_b}{2}\right) - A_{dc}\left(\frac{v_a + v_b}{2}\right) \tag{1.20}$$

The importance of the existence of the transfer gains may be appreciated by reconsideration of the previous numerical example, in which $R_L = 10$ k, $R = 10$ k, $r_p = 10$ k, and $\mu = 20$. Suppose in addition that the amplification factor of the right-hand tube exceeds that of the left-hand tube to such a degree that $\delta\mu/\mu = -0.1$. The four gain parameters then have the following values:

$$A_{cc} = 0.455 \tag{1.21}$$

$$A_{dd} = 10(1 + 0.0091) \tag{1.22}$$

$$A_{cd} = -1 \tag{1.23}$$

$$A_{dc} = -0.09 \tag{1.24}$$

Substitution of these values into Eq. 1.20 shows that a given DM output voltage can be produced not only from a DM input voltage, but also from a CM input voltage $A_{dd}/A_{dc} = 110$ times as large in magnitude. Hence a 0.11-v CM input signal is indistinguishable from a 1-mv DM input signal,

as far as the floating output is concerned. It should be recalled that if the circuit were balanced, there would be *no* output between the floating terminals due to a common-mode input signal. Moreover, if a single-ended output is employed, so that the output voltage of interest is v_{1a} or v_{1b}, then the presence of unbalanced tube amplification factors leads to an even more serious dependence on the CM input voltage. By addition and subtraction of Eqs. 1.19 and 1.20,

$$v_{1a} = -(A_{dd} + A_{cd})\left(\frac{v_a - v_b}{2}\right) - (A_{cc} + A_{dc})\left(\frac{v_a + v_b}{2}\right) \tag{1.25}$$

$$v_{1b} = -(A_{dd} - A_{cd})\left(\frac{v_a - v_b}{2}\right) - (A_{cc} - A_{dc})\left(\frac{v_a + v_b}{2}\right) \tag{1.26}$$

and hence, for the output v_{1a}, a DM input voltage is indistinguishable from a CM input voltage $(A_{dd} + A_{cd})/(A_{cc} + A_{dc}) = 24.7$ times as large.

The degree to which a differential amplifier can prevent an output arising from a CM input is perhaps its most important property, and is described by the "common-mode rejection factor." The common-mode rejection factor is defined as the ratio of the CM input voltage to the DM input voltage that give rise to the same output voltage. As may be seen from our examples, the CM rejection factor depends on which output voltage is considered. If the floating output, where $(v_{1a} - v_{1b})/2$ is the output voltage of interest, is employed, then from Eq. 1.20 the CM rejection factor is A_{dd}/A_{dc}; if single-ended output, where v_{1a} is the output voltage of interest, is employed, then from Eq. 1.25 the CM rejection factor is $(A_{dd} + A_{cd})/(A_{cc} + A_{dc})$.

The CM rejection factor should be carefully distinguished from the discrimination factor A_{dd}/A_{cc}. As pointed out by Parnum,[5] a high value of discrimination factor is not sufficient to ensure a high value of CM rejection factor. Parnum noted that although an arbitrarily high discrimination factor can be achieved by using a high effective value of cathode coupling resistance, the CM rejection factor under the same conditions becomes asymptotic to a limiting finite value determined by the circuit unbalances. Thus, if a constant-current pentode is used as the cathode coupling impedance as in the circuit of Fig. 7, the effective value of R becomes very large and the two gains A_{dd} and A_{dc}, from Eqs. 1.16 and 1.18, become asymptotic to

$$A_{dd} = \frac{\mu R_L}{r_p + R_L}\left[1 - \left(\frac{\delta\mu}{\mu}\right)^2 \frac{\mu}{1+\mu}\right] \approx \frac{\mu R_L}{r_p + R_L} \tag{1.27}$$

$$A_{dc} = \frac{\delta\mu}{\mu}\frac{\mu R_L}{r_p + R_L}\frac{1}{1+\mu} \tag{1.28}$$

The CM rejection factor with respect to the floating output, A_{dd}/A_{dc}, thus approaches the limiting value $\mu(1 + \mu)/\delta\mu$ even though the discrimination factor A_{dd}/A_{cc} approaches infinity. For the typical figures chosen in the previous example, this limiting magnitude of the CM rejection factor is only 210, a value much too low to provide adequate performance in the applications described in Section 1.1. Some methods of adjusting the circuit to compensate for unbalances, and thereby increase the CM rejection factor, have been suggested by Parnum in a later paper.[6]

In the period between these two articles, the combination of Offner's CM negative feedback with Goldberg's pentode cathode impedance had been proposed[7] and was shown to give appreciably higher values of discrimination factor. The principal benefit of CM negative feedback is to increase the effective value of the cathode coupling resistance, although we have seen that in spite of an essentially infinite value of R an unbalance in the tube amplification factors imposes an upper limit on the CM rejection factor. Unbalances in other parts of the differential-amplifier circuit, however, not included in our analysis, also contribute to a noninfinite CM rejection factor, and these contributions do not reach limiting values when R tends to infinity (see Appendix II). Application of CM negative feedback therefore not only improves the discrimination factor, but is also of considerable value in improving the CM rejection factor.

There are yet other properties of a differential amplifier, some of which may be affected by CM negative feedback. For example, Offner[8] has recognized that unbalances in the circuit symmetry could permit the inverse cross-coupling effect, namely, that a CM output could be developed from a DM input. This property is apparent through the presence of the $\delta\mu/\mu$ term in Eq. 1.13. Later work[9–11] has developed refinements in the theory and application, removed inconsistencies, and provided more detailed interpretation of the properties of vacuum-tube differential amplifiers. Understanding of the use of the basic differential stage as a push-pull amplifier also developed concurrently, and Birt[12] has recently presented an excellent discussion of the qualitative effects of CM negative feedback on the balance properties of multistage push-pull amplifiers.

The use of transistors in differential amplifiers has also received attention, though of much less detail than that enjoyed by tube circuits. The recent developments in transistor direct-coupled d-c amplifiers[13,14] have stimulated renewed interest in differential-amplifier circuits, and the stringent performance requirements have dictated development of analysis methods which can accommodate the critical effects of unbalances in the circuit symmetry.

It is apparent from the examples already discussed that the properties of differential amplifiers are considerably more complex than those of ordinary, single-ended amplifiers, and that a number of additional parameters are required to describe their performance. It has been shown that some of these performance parameters are strongly dependent on circuit unbalances; and that, unfortunately, analysis by conventional methods of the effects of unbalances rapidly becomes extremely complicated. The algebra leading to Eqs. 1.13 and 1.14 is fairly lengthy, and yet these results apply only to the simplest form of differential amplifier (shown in Fig. 5); only a-c signals were considered, and unbalance in only one pair of elements was taken into account. Furthermore, since a vacuum-tube circuit was the one discussed, the input impedances were essentially infinite; in transistor circuits this is not so. It was seen that the system requirements of encephalography or strain-gauge instrumentation include high input impedances to the differential amplifier, and we have not as yet defined the "input impedance" of a floating-input differential amplifier, nor considered the possible effects of circuit unbalances on this impedance.

Obviously some alternative analysis approach must be developed to handle multistage differential amplifiers, to include d-c signals, to include CM and possibly also DM negative feedback, and particularly to include the effects of many more unbalances in symmetrical elements. A suitable analysis method is developed in the following chapters; the properties of differential-amplifier circuits are established and summarized in an integrated manner, and application of the analysis method to particular transistor d-c amplifier circuits is presented in examples.

In the final section of this chapter the nature of the problem is reviewed, and the steps to be followed for its solution are outlined.

1.4 The Nature and Treatment of the Analysis of Unbalanced Symmetrical Circuits

It is usually desired that a symmetrical circuit be balanced, that is, that homologous elements be equal in magnitude. In this special case, there is available a simple and straightforward analysis technique whereby use of the bisection theorem permits complete results to be obtained from consideration of only one half of the symmetrical circuit.[15] This technique, reviewed in Chapter 2, rests on the division of arbitrary currents and voltages in the two halves of the circuit into CM and DM components. The performance of the half-circuit in response to each component is then calculated separately, and the complete performance is obtained by superposition of the two independent results.

An exactly balanced symmetrical circuit is an idealization that cannot be achieved in practice. There will always be unavoidable unbalances due to

the finite tolerances of the component values. Any active devices used will be the biggest offenders in this respect, although even the contribution of precision resistors may be significant. Unfortunately, quite small unbalances may have far-reaching effects on the performance of the circuit, the most serious of which is the resulting interaction between the CM and DM components of the signal. In particular, a CM input signal produces a DM output signal indistinguishable from one produced by a DM input signal and thus gives rise to errors. This undesirable effect is described quantitatively by the CM rejection factor, which ideally should be infinite, and indeed is so in an exactly balanced symmetrical circuit. In symmetrical d-c amplifier circuits, the equivalent input drift may also be significantly affected by unbalances.

The advantages of the bisection theorem are generally no longer available when the two halves of the symmetrical circuit are not balanced, and analysis of the complete circuit must be undertaken with different symbols attached to each element of a homologous pair. The result of this process, as given by Slaughter,[16] for example, is enough to discourage further attempts along these lines. However, if the unbalance between homologous elements is a small percentage of their average value, some simplifications may be effected even though the circuit is still treated as a whole. Sporadic interest in this approach[8–11] has led to some useful results, including analytical expressions for the CM rejection factor. The analysis nevertheless becomes completely unwieldy for all but the simplest circuits.

An analysis technique which treats small-percentage unbalances by an extension of the method using the bisection theorem is developed inductively in Chapter 2 from the specific example of a transistor emitter-coupled d-c differential amplifier. The technique may be called a "sequential" method of solution since the analysis is divided into a succession of individually simple steps. It is shown how internal dependent and independent generators, and their unbalances, may be incorporated in the analysis. A more general and rigorous establishment of the technique is given in Appendix I.

In Chapter 3 the performance properties of a differential amplifier are defined, and the sequential method of analysis developed in Chapter 2 is carried through to completion for the transistor d-c differential amplifier. Analytical expressions are obtained for performance quantities which are directly related to the circuit unbalances, such as the CM rejection factor and the transfer gains (CM input to DM output, and vice versa). The various amplifier supply voltages constitute CM inputs and can be just as effective in causing equivalent differential input signals as can a normal CM input signal; hence power supply rejection factors are defined and expressions for them obtained. It is shown that the input to the amplifier may be

represented by a three-terminal, y-parameter network plus CM and DM equivalent-input drift current generators, where the four admittances are identified as the CM and DM input admittances and two transfer input admittances. We also show how a differential signal source, such as a bridge, may be represented by a three-terminal z-parameter network plus CM and DM equivalent source voltages, where the four impedances are identified as the CM and DM source impedances and two transfer source impedances related to source unbalances.

Typical numerical values are employed in an example of the overall performance of a differential amplifier including the signal source, in which we show how the various performance quantities depend on the circuit unbalances. The results show that the CM rejection factor is seriously degraded by small-percentage unbalances, and that balanced source resistances in conjunction with nonzero transfer input admittances can be as effective in degrading the CM rejection factor as can unbalanced source resistances in conjunction with the CM and DM input admittances.

In Chapter 4 a modification of the basic emitter-coupled differential amplifier is discussed, in which a constant-current transistor placed in the common emitter path is shown to be capable of improving the CM rejection factor. In Chapter 5 the very considerable advantages that accrue from the use of CM negative feedback are discussed, and analytical and typical numerical results are given for a two-stage differential amplifier of practical importance. However, the advantages of CM feedback are not entirely unmixed blessings, and detailed attention in any particular case should be given to its effect on the equivalent differential input drift and the power supply rejection factors.

The sequential analysis method we develop is simple enough for the more complex unbalanced symmetrical circuits discussed in Chapters 4 and 5 to be reasonably handled. Not only is greater insight obtained into the results of unbalances; quantitative expressions of adequate simplicity for practical use can be derived for these important effects. Although our treatment is entirely in terms of transistor direct-coupled d-c differential amplifiers, in which modern transistors allow performance vastly superior to the earlier, the method is equally applicable to vacuum-tube differential amplifiers and to any symmetrical circuit in which the effects of small-percentage unbalances are important. The method can also be used to determine the effects of unequal distortion generation in the two sides of a push-pull amplifier, and can be extended to analyze the properties of partly unsymmetric circuits. Examples are d-c regulated power supplies which contain a differential stage as an error detector, and differential-input, single-ended-output a-c and d-c amplifiers.

Chapter Two

Development of a sequential method for analysis of unbalanced symmetrical circuits

In this chapter a sequential method of analyzing symmetrical circuits with small unbalances is inductively developed with reference to a particular circuit. A formal proof of the method is given in Appendix I. First, an analysis method applicable to balanced symmetrical circuits will be reviewed.

2.1 Use of the Bisection Theorem for Analysis of Balanced Symmetrical Circuits

An example of a symmetrical amplifier circuit is shown in Fig. 8. Specifically, the circuit represents a transistor emitter-coupled d-c differential amplifier and is used as the basis for development of the method for treating unbalances in symmetrical circuits. We choose to discuss d-c performance, since unbalances are particularly important in d-c amplifiers, but of course the results are equally valid for a-c conditions.

If the two transistors in Fig. 8 are identical, and if $R_{1a} = R_{1b}$ and $R_{2a} = R_{2b}$, the circuit is balanced and the bisection theorem[15] can be applied. We divide the circuit of Fig. 8 along its vertical line of symmetry and draw two different equivalent circuits for one of the sides, say side *a*. One equivalent circuit, the *common-mode equivalent half-circuit*, is valid when the input signals v_a and v_b to the two sides *a* and *b* are equal in magnitude and phase; the other, the *differential-mode equivalent half-circuit*, is valid when the two input signals are equal in magnitude but opposite in phase. The two equivalent half-circuits are shown in Fig. 9; they are established as follows.

For a CM input signal $v_a = v_b$, the signals at homologous points in Fig.

8 are equal since the circuit is balanced. Hence there is no current in elements that connect homologous points, that is, in elements that are topologically perpendicular to the line of symmetry, such as R_4. Elements that lie along the topological line of symmetry, such as R_3, carry a current twice that present in one side. Thus in the CM equivalent half-circuit, elements such as R_4 are open circuited and elements such as R_3 appear with twice their actual value.

For a DM input signal $v_a = -v_b$, the signals at homologous points in Fig. 8 are equal in magnitude but opposite in phase, since the circuit is balanced. Hence a virtual ground exists at the center of elements such as R_4, and there is no current in elements such as R_3. Thus in the DM equivalent half-circuit, elements such as R_4 appear with half their actual value and are returned to ground, and elements such as R_3 are short-circuited. In addition, supply voltages such as E_1 and E_2 are short-circuited since they are CM voltages and do not introduce DM signals.

The model used in Fig. 9 for the transistors is a very simple one. Internal emitter resistance is assumed to be negligibly small, or can be considered lumped with the external emitter resistance R_1; internal base resistance is assumed to be negligibly small, or can be considered lumped with the signal source resistance which will be introduced later; and internal collector resistance is assumed to be negligibly large. All that remains is a common-emitter T circuit, for which the collector current is equal to β_1 times the base current plus a saturation current, and the base-emitter voltage is the internal diode drop. The common-emitter T is preferred[17] over the common-base T, and it should be noted that the saturation current is the open-base and not the open-emitter collector current (I_{c0}). In circuit diagrams, dependent current generators are distinguished from independent ones by use of a square instead of a circle.

The two equivalent half-circuits of Fig. 9 permit straightforward analysis of the performance for CM or for DM input signals. By superposition, results can be obtained for arbitrary input voltages v_a and v_b, since they can always be divided into CM and DM components v_c and v_d, defined as

$$v_c = \frac{v_a + v_b}{2} \tag{2.1}$$

$$v_d = \frac{v_a - v_b}{2} \tag{2.2}$$

Thus a balanced symmetrical circuit can be analyzed simply. It should be emphasized that although the circuit element values must be balanced for this method to be valid, internal independent generators need not be balanced; that is, although the two transistors of Fig. 8 must have equal β's,

they need not have equal saturation currents or base-emitter voltage drops. From an analysis point of view, these internal generators are indistinguishable from external generators, and arbitrary values on the two sides can therefore be represented by CM and DM components. Thus if the saturation currents are I_{1a} and I_{1b} and the base-emitter voltages are V_{1a} and V_{1b}, the quantities to be shown in the two equivalent half-circuits are

$$I_{1c} = \frac{I_{1a} + I_{1b}}{2} \tag{2.3}$$

$$I_{1d} = \frac{I_{1a} - I_{1b}}{2} \tag{2.4}$$

$$V_{1c} = \frac{V_{1a} + V_{1b}}{2} \tag{2.5}$$

$$V_{1d} = \frac{V_{1a} - V_{1b}}{2} \tag{2.6}$$

Of course, if the two transistors are identical in all respects, the independent generators I_{1d} and V_{1d} in Fig. 9*b* will each be zero.

2.2 Analysis of Unbalanced Symmetrical Circuits by Extension of the Bisection Theorem Method

We now wish to extend the bisection theorem procedure to cover situations where circuit unbalances exist, that is, where $\beta_{1a} \neq \beta_{1b}$, $R_{1a} \neq R_{1b}$, etc. To set up such a method, we consider the physical operation of the circuit of Fig. 8. Suppose first that all homologous elements are balanced except that $R_{2a} \neq R_{2b}$. If a pure CM input signal $v_a = v_b = v_c$ is applied to the inputs, the currents at the two transistor collectors will be equal. The collector voltages will not be equal, however, which means that a differential component has been introduced by the unbalance between R_{2a} and R_{2b}. Thus a cross-coupling effect, whereby a CM input signal gives rise to a DM output, is introduced. In general, therefore, it is no longer possible to draw independent CM and DM half-circuits. However, if the unbalance is small, the CM half-circuit and currents and voltages will be little affected, and the resistance R_2 may be taken to represent the average of R_{2a} and R_{2b}. At the same time, the result of the unbalance may be represented in the DM half-circuit by a voltage generator in series with R_2 whose value is proportional to the unbalance in R_{2a} and R_{2b} and to the CM current which flows in R_2. By an analogous argument, if a pure DM input

signal $v_a = -v_b = v_d$ is applied to the inputs, a similar cross-coupling effect, whereby a DM input signal gives rise to a CM output, is introduced. The DM half-circuit and currents and voltages will be little affected by small unbalances in R_{2a} and R_{2b}, and the resistance R_2 in Fig. 9*b* may again be taken to represent their average value. The result of the unbalance may be represented in the CM half-circuit by a voltage generator in series with R_2 whose value is proportional to the unbalance in R_{2a} and R_{2b} and to the DM current which flows in R_2. A similar procedure may be followed for unbalances in any other homologous elements. Unbalance in the transistor β's may be accounted for by introducing an additional current generator into the CM half-circuit whose value is proportional to the unbalance in the β's and to the DM base current, and similarly for the DM half-circuit.

The addition of the various *interaction* generators, represented by shaded circles, is shown in Fig. 10. An expression for the magnitude of each interaction generator is obtained as follows. Assign a symbol to the CM and DM current components in each element, for example i_{2c} and i_{2d} in R_2. If $R_{2a} = R_2 + \delta R_2$ and $R_{2b} = R_2 - \delta R_2$, then the interaction generators to be placed in the CM and DM half-circuits are $\delta R_2 i_{2d}$ and $\delta R_2 i_{2c}$ respectively, with the polarities shown in Fig. 10. Similarly, if $\beta_{1a} = \beta_1 + \delta\beta_1$ and $\beta_{1b} = \beta_1 - \delta\beta_1$, the corresponding interaction generators are $\delta\beta_1 i_d$ and $\delta\beta_1 i_c$. Now although the interaction generators are controlled, they are independent generators as far as separate analysis of each half-circuit is concerned, since each is dependent only on currents in the other half-circuit. Hence solution for the complete performance of the circuit, in terms of the interaction generators, can be obtained by superposition of the results for each half-circuit. In general, simultaneous solution of the two sets of equations will be required to eliminate the interaction generator terms from the result. This procedure is as complicated as a general analysis of the circuit of Fig. 8 would be, and hence it appears that little is gained by invoking the bisection theorem method. However, if the elements are unbalanced by only small percentages, the CM and DM currents and voltages will be little different from what they would be if the circuit were balanced, and hence the interaction generators can be expressed in terms, not of the actual currents, but of the "original" currents which would be present in the absence of unbalances. The approximation involved here is that third-order interaction terms are neglected.

The significance of the extra subscript *o* in Figs. 9 and 10 now becomes apparent. In Fig. 9 the circuit is assumed balanced, and so all currents and voltages are given the extra subscript *o*; in Fig. 10 the circuit is assumed unbalanced by only small percentages so that the interaction generators may be expressed in terms of the "original" currents rather than of the "actual" ones which do not have the subscript *o*. This approximation eliminates the

necessity for simultaneous solution of both half-circuits, since the analysis may be completed in independent stages. In the vacuum-tube example discussed in Chapter 1, the approximation permits omission of the $(\delta\mu/\mu)^2$ term in Eqs. 1.16 and 1.27.

The method for analyzing symmetrical circuits with small-percentage unbalances may be summarized as follows. Draw the CM and DM half-circuits assuming that the circuit elements are balanced, using the average value for each pair of homologous elements, but retain any unbalances in the internal independent generators. Assign symbols with subscript o to all signals of interest, including those present in each element whose unbalance is to be incorporated. Calculate all these signals in terms of the internal and external independent generators. Next, draw the modified half-circuit including the interaction generators, and calculate the desired performance quantities in terms of the internal and external independent generators and of the "independent" interaction generators. Finally, eliminate the interaction generator terms by substitution for quantities previously calculated in the "original" half-circuits.

This method permits solution of unbalanced symmetrical circuits by breaking down the analysis into a succession of individually simple steps. A more rigorous derivation of the general method is given in Appendix I. The "brute force" approach, that of analyzing the complete unbalanced circuit as a whole, would become almost impossibly complicated for practical use, even for a circuit apparently as simple as that of Fig. 8. The sequential method here developed is useful for still more complicated circuits, but care must nevertheless be taken that the algebra does not get out of hand. The following chapters illustrate the practical application of the sequential method, first for the circuit of Fig. 8, and then for more complicated arrangements.

Chapter Three

Application of the sequential method to the analysis of a single-stage transistor d-c differential amplifier

The method outlined in the previous chapter for the analysis of a transistor d-c differential amplifier will now be followed through. The work is divided into four parts. In the first, the analysis goals are established and definitions given of the various quantities which describe the amplifier performance, and it is shown how conditions at the input terminals can be represented by four short-circuit admittances and two current generators. In the second part, the analysis is carried out to obtain these performance parameters. In the third part, the representation of a signal source by four open-circuit impedances and two voltage generators is developed, and the conditions at the amplifier input when fed from such a source are established. In the fourth part, the overall performance parameters of the amplifier and signal source are obtained. It is convenient to separate the source from the amplifier in this way, rather than to analyze the combination as a unit, since a given amplifier is frequently used with different sources, and it is desirable to isolate effects due to the source.

3.1 Definitions of the Performance Parameters of a Differential Amplifier

The circuit to be discussed is shown in Fig. 8. The signal source impedances have already been assumed to be zero in the equivalent circuits of Figs. 9 and 10. Several quantities of interest describe the performance of this circuit and must be defined before the analysis is begun.

The performance parameters fall into two groups: *First-order* quantities, which describe properties of the *balanced* circuit, and *second-order* quantities,

which describe properties arising from circuit *unbalances.* Third-order quantities, which arise from interaction between second-order quantities, will be neglected, as described in the previous chapter.

Both the common-mode (CM) and the differential-mode (DM) output voltages are of interest. From Fig. 10*a* we see that the CM output voltage v_{1c} contains components arising from several sources: First-order components arise from the (external) CM input voltage v_c, the two (external) supply voltages E_1 and E_2, and the two (internal) CM sources V_{1c} and I_{1c}; second-order components arise through the interaction generators from the (external) DM input voltage v_d and the two (internal) DM sources V_{1d} and I_{1d}. From Fig. 10*b* it is seen that the DM output voltage v_{1d} contains first-order components produced by the (external) DM input voltage v_d and the two (internal) DM sources V_{1d} and I_{1d}, and second-order components arising from v_c, E_1, E_2, V_{1c}, and I_{1c}. The output properties of the circuit of Fig. 8 can thus be described by two equations, one for the CM component v_{1c} and one for the DM component v_{1d} of the output voltage:

$$v_{1c} = -A_{cc}\left(v_c + \frac{1}{H_d}v_d + V_{ci} + \frac{1}{A_1}E_1 + \frac{1}{A_2}E_2\right) \qquad (3.1)$$

$$v_{1d} = -A_{dd}\left(v_d + \frac{1}{H_c}v_c + V_{di} + \frac{1}{H_1}E_1 + \frac{1}{H_2}E_2\right) \qquad (3.2)$$

The coefficients A_{cc} and A_{dd} are, respectively, the *CM gain* and the *DM gain* (the minus sign is explicit in order to make A_{cc} and A_{dd} positive quantities). The term v_d/H_d represents the CM equivalent input voltage resulting from the DM signal v_d. Thus H_d is defined as the *DM rejection factor,* and is a second-order quantity which becomes infinite when the circuit is balanced. The quantity $-A_{cc}v_d/H_d$ is the CM output voltage produced by a DM input voltage v_d, and thus $A_{cd} \equiv A_{cc}/H_d$ is defined as the *DM-to-CM transfer gain,* which is also a second-order quantity, and becomes zero when the circuit is balanced. In a similar manner, H_c is defined as the *CM rejection factor* and $A_{dc} \equiv A_{dd}/H_c$ as the *CM-to-DM transfer gain.* The quantities V_{ci} and V_{di} are respectively the *CM* and *DM equivalent input voltages due to the internal independent sources,* and each contains both first-order and second-order terms. The quantities

$$V_{ce} \equiv \frac{1}{A_1}E_1 + \frac{1}{A_2}E_2 \qquad (3.3)$$

$$V_{de} \equiv \frac{1}{H_1}E_1 + \frac{1}{H_2}E_2 \qquad (3.4)$$

are respectively the *CM* and *DM equivalent input voltages produced by the external sources other than the input signals v_c and v_d,* where A_1 and A_2 are first-order quantities, and H_1 and H_2 are second-order quantities which both become infinite when the circuit is balanced. By analogy with the definition of H_c, the quantities H_1 and H_2 are defined as the *power-supply (PS) rejection factors.*

It is usually desirable to make the CM gain A_{cc} as small as possible compared with the DM gain A_{dd}, since normally it is required to amplify DM to the exclusion of CM signals. A quantity defined as the *discrimination factor* $F \equiv A_{dd}/A_{cc}$ is used to describe the gain ratio, where F should be large. Another important quantity is the *output fractional unbalance* $U \equiv A_{cc}/A_{dd}H_d = 1/FH_d$, which is the ratio of the CM to DM output voltage produced by a DM input voltage. For an exactly balanced circuit, U is zero. Perhaps the most important performance quantities are the CM and PS rejection factors H_c, H_1, H_2, and the DM equivalent input voltage V_{di}. These quantities describe spurious DM output voltages that do not arise from the DM input signal, and that should be minimized since they are extraneous. The first-order components of V_{di} arise from inequalities in the transistor independent generators and are therefore directly related to the degree of mismatch of the two transistors. The second-order components of V_{di} and the three H's are all produced by the interaction generators, and hence they are directly related to the circuit unbalances.

In the literature on differential amplifiers there has been a certain lack of unanimity on the names of the various performance parameters, which has been due in part to some confusion about their meaning. For example, it was not always recognized that a CM output could be developed by a DM as well as by a CM input, and hence the terms "discrimination" and "rejection" have sometimes been used rather loosely.[9] Although the "discrimination factor" and the symbol F are now quite firmly established as representing the DM-to-CM gain ratio, the "CM rejection factor" is sometimes called the "transmission factor."[6] We feel that "rejection" is a more appropriate description than "transmission," since an infinite value of this factor corresponds to infinite rejection of CM signals (as far as their interaction effect is concerned). Although the symbol H is established for a rejection factor, it has not previously been given a distinguishing subscript, because the existence of finite PS as well as CM rejection factors appears not to have been noted. Various names and symbols have also been applied to the several gain parameters. For example, the DM (antiphase) gain A_{dd} has been represented by M (Parnum[5]), G_0 (Offner[8]), and α (Klein[11]); the CM (inphase) gain by $\bar{M}$, G_c, and δ; and the CM-to-DM transfer gain A_{dc} by K, G_i, and β (it is also called the "inversion gain" by Offner[8]). The DM-to-CM transfer gain A_{cd} has been represented by G_u (called the

"differential unbalance" by Offner[8]), and by γ (Klein[11]). The existence of the DM rejection factor H_d was also recognized by Klein, and given the symbol G. In the light of the multiplicity of names and symbols previously applied to the various performance parameters, it is hoped that the symbols and subscripts chosen here will provide a logically developed and complete set that emphasizes the symmetry between the CM and the DM circuit performance.

The performance parameters so far introduced all relate to the gain properties of the amplifier circuit. Since it is desired to drive the amplifier from an arbitrary signal source, it is also necessary to define quantities that describe the input properties of the amplifier. From Fig. 10 we see that the CM and DM input currents i_c and i_d contain components due to several sources. Remarks similar to those for the two output voltages apply, and the input properties of the circuit of Fig. 8 can be described by two equations, one for the CM component i_c and one for the DM component i_d of the input currents:

$$i_c = y_{cc}v_c + y_{cd}v_d + I_c \tag{3.5}$$

$$i_d = y_{dc}v_c + y_{dd}v_d + I_d \tag{3.6}$$

where y_{cc} and y_{dd} are respectively the *CM* and *DM input admittances*, and are first-order quantities. The terms in y_{cd} and y_{dc} arise from the interaction generators and are defined respectively as the *DM-to-CM* and *CM-to-DM input transfer admittances*. These are both second-order quantities and are zero in a balanced circuit. The quantities I_c and I_d are, respectively, the *CM* and *DM input currents* arising from all sources other than the CM and DM input signals v_c and v_d, and they may be expressed as

$$I_c = I_{ci} + I_{ce} \tag{3.7}$$

$$I_d = I_{di} + I_{de} \tag{3.8}$$

where the components produced by internal and external sources are distinguished by subscripts i and e respectively. The internal sources are again V_{1c}, V_{1d}, etc., and the external sources are E_1 and E_2.

Solution of the differential-amplifier circuit of Fig. 8 involves analysis to find expressions for the various performance parameters we have introduced. The performance parameters are obtained from equations for the CM and DM output voltages v_{1c} and v_{1d}, and for the CM and DM input currents i_c and i_d, in terms of the independent generators. It is useful to summarize the definitions of the performance parameters in tabular form, both in words and in the formulas from which the expressions are derived. To permit identification of the terms required in a particular derivation, the notation $v_{1c}[E_1]$ indicates that component of v_{1c} which is produced by E_1, $i_d[V_{1d}, I_{1d}]$ indicates the components of i_d that arise from V_{1d} and I_{1d}, and similarly for other quantities. The collected definitions are displayed in Table I.

Application of the sequential analysis method to the solution of the circuit of Fig. 8, which leads to expressions for these performance parameters, will be discussed next.

3.2 Amplifier Analysis with Zero Signal Source Impedance

The application of the sequential method to the analysis of the transistor d-c differential-amplifier circuit of Fig. 8, summarized at the end of Chapter 2, involves separate solution of the four circuits in Figs. 9 and 10. It is clear that there will be a considerable amount of duplication of effort, since these four circuits differ in only minor respects. Before embarking on the solution of any problem it is always worthwhile to consider how the algebraic manipulations may be minimized, and a more efficient procedure for the present problem is now suggested.

First, since Figs. 9*a* and 9*b* are merely special cases of Figs. 10*a* and 10*b*, only the general cases of Fig. 10 need be analyzed. Further, since Figs. 10*a* and 10*b* differ in form only in the presence or absence of certain components, effort will be saved if we analyze a composite circuit that contains all elements present in both the CM and DM half-circuits. This *composite equivalent half-circuit* is shown in Fig. 11, and contains the additional simplification that parallel or series generators are combined and a single symbol, e or i, is used to represent each internal voltage or current generator. It is now necessary to analyze only the circuit of Fig. 11; then results can be obtained for each of the four circuits of Figs. 9 and 10 merely by making appropriate substitutions and restrictions.

Solution of the circuit of Fig. 11 implies the derivation of expressions for the four currents i, i_1, i_2, and i_3 in terms of the various generators. By straightforward analysis, these are

$$i = \frac{v + E_2 - e_1 - (R_1 + 2R_3)j_1}{(1 + \beta_1)(R_1 + 2R_3)} \tag{3.9}$$

$$i_1 = \frac{v + E_2 - e_1}{R_1 + 2R_3} \tag{3.10}$$

$$i_2 = \frac{E_1 - e_2}{R_2 + R_4/2} + \frac{\alpha_1 R_4/2}{R_2 + R_4/2}\left(\frac{v + E_2 - e_1}{R_1 + 2R_3}\right) + \frac{R_4/2}{R_2 + R_4/2}\,\frac{j_1}{1 + \beta_1} \tag{3.11}$$

$$i_3 = \frac{E_1 - e_2}{R_2 + R_4/2} - \frac{\alpha_1 R_2}{R_2 + R_4/2}\left(\frac{v + E_2 - e_1}{R_1 + 2R_3}\right) - \frac{R_2}{R_2 + R_4/2}\,\frac{j_1}{1 + \beta_1} \tag{3.12}$$

where $\alpha_1 \equiv \beta_1/(1 + \beta_1)$ is the average common-base current gain of the two transistors. The output voltage v_1 can be found as

$$v_1 = \frac{R_4}{2} i_3 \tag{3.13}$$

These results may now be made applicable to the circuits of Figs. 9 and 10 by appropriate modifications. For the half-circuits of Fig. 10, only the input current and output voltage are required since these are sufficient to determine the performance parameters. For Fig. 10*a*, these are obtained from Eqs. 3.9 and 3.13 by adding the subscript c, by setting $R_4 = \infty$, and by substituting $e_1 = V_{1c} + \delta R_1 i_{1do}$, $e_2 = \delta R_2 i_{2do}$, $j_1 = I_{1c} + \delta\beta_1 i_{do}$:

$$i_c = \frac{v_c + E_2 - V_{1c} - \delta R_1 i_{do} - (R_1 + 2R_3)(I_{1c} + \delta\beta_1 i_{do})}{(1 + \beta_1)(R_1 + 2R_3)} \tag{3.14}$$

$$v_{1c} = E_1 - \delta R_2 i_{2do} - \alpha_1 R_2\left(\frac{v_c + E_2 - V_{1c} - \delta R_1 i_{1do}}{R_1 + 2R_3}\right) - R_2\frac{I_{1c} + \delta\beta_1 i_{do}}{1 + \beta_1} \tag{3.15}$$

For Fig. 10*b*, these are obtained from Eqs. 3.9 and 3.13 by adding the subscript d, by setting $R_3 = 0$, $E_1 = E_2 = 0$, and by substituting $e_1 = V_{1d} + \delta R_1 i_{1co}$, $e_2 = \delta R_2 i_{2co}$, $j_1 = I_{1d} + \delta\beta_1 i_{co}$:

$$i_d = \frac{v_d - V_{1d} - \delta R_1 i_{1co} - R_1(I_{1d} + \delta\beta_1 i_{co})}{(1 + \beta_1)R_1} \tag{3.16}$$

$$v_{1d} = \frac{R_4/2}{R_2 + R_4/2}\left[-\delta R_2 i_{2co} - \alpha_1 R_2\left(\frac{v_d - V_{1d} - \delta R_1 i_{1co}}{R_1}\right) - R_2\frac{I_{1d} + \delta\beta_1 i_{co}}{1 + \beta_1}\right] \tag{3.17}$$

Equations 3.14 through 3.17 contain implicitly all the information needed to obtain the performance quantities defined in Table I. However, the terms arising from the interaction generators must first be eliminated by making Eqs. 3.9 through 3.11 applicable to the balanced half-circuits of Fig. 9, in which the interaction generators are zero. For Fig. 9*a* this is accomplished by adding the subscript *co*, setting $R_4 = \infty$, and substituting $e_1 = V_{1c}$, $e_2 = 0$, $j_1 = I_{1c}$:

$$i_{co} = \frac{v_c + E_2 - V_{1c} - (R_1 + 2R_3)I_{1c}}{R_1 + 2R_3} \tag{3.18}$$

$$i_{1co} = \frac{v_c + E_2 - V_{1c}}{R_1 + 2R_3} \tag{3.19}$$

$$i_{2co} = \alpha_1\left(\frac{v_c + E_2 - V_{1c}}{R_1 + 2R_3}\right) + \frac{I_{1c}}{1 + \beta_1} \tag{3.20}$$

For Fig. 9*b* this is accomplished by adding the subscript *do*, setting $R_3 = 0$, $E_1 = E_2 = 0$, and substituting $e_1 = V_{1d}$, $e_2 = 0$, $j_1 = I_{1d}$:

$$i_{do} = \frac{v_d - V_{1d} - R_1 I_{1d}}{(1 + \beta_1)R_1} \tag{3.21}$$

$$i_{1do} = \frac{v_d - V_{1d}}{R_1} \tag{3.22}$$

$$i_{2do} = \frac{R_4/2}{R_2 + R_4/2}\left(\frac{v_d - V_{1d}}{R_1} + \frac{I_{1d}}{1 + \beta_1}\right) \tag{3.23}$$

Substitution of Eqs. 3.18 through 3.23 into Eqs. 3.15 and 3.17 and rearrangement of terms shows that the CM and DM output voltages may be expressed in the forms stated in Eqs. 3.1 through 3.4, where

$$A_{cc} = \frac{\alpha_1 R_2}{R_1 + 2R_3} \tag{3.24}$$

$$A_{dd} = \frac{\alpha_1}{R_1}\,\frac{R_2 R_4/2}{R_2 + R_4/2} \tag{3.25}$$

$$\frac{1}{H_d} = -\frac{\delta R_1}{R_1} + \frac{R_1 + 2R_3}{R_1}\left(\frac{1}{\alpha_1}\,\frac{R_4/2}{R_2 + R_4/2}\,\frac{\delta R_2}{R_2} + \frac{1}{1 + \beta_1}\,\frac{\delta\beta_1}{\beta_1}\right) \tag{3.26}$$

$$\frac{1}{H_c} = \frac{R_1}{R_1 + 2R_3}\left(\frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} + \frac{1}{1 + \beta_1}\,\frac{\delta\beta_1}{\beta_1}\right) \tag{3.27}$$

$$\frac{1}{A_1} = -\frac{R_1 + 2R_3}{\alpha_1 R_2} \tag{3.28}$$

$$\frac{1}{A_2} = 1 \tag{3.29}$$

$$\frac{1}{H_1} = 0 \tag{3.30}$$

$$\frac{1}{H_2} = \frac{1}{H_c} \tag{3.31}$$

$$V_{ci} = -V_{1c} + (R_1 + 2R_3)\frac{I_{1c}}{\beta_1}$$
$$+ \left[\frac{\delta R_1}{R_1} - \frac{R_1 + 2R_3}{R_1}\left(\frac{1}{\alpha_1}\frac{R_4/2}{R_2 + R_4/2}\frac{\delta R_2}{R_2} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right)\right] V_{1d}$$
$$+ (R_1 + 2R_3)\left(\frac{1}{\alpha_1}\frac{R_4/2}{R_2 + R_4/2}\frac{\delta R_2}{R_2} - \frac{\delta\beta_1}{\beta_1}\right)\frac{I_{1d}}{1+\beta_1} \quad (3.32)$$

$$V_{di} = -V_{1d} + R_1\frac{I_{1d}}{\beta_1} - \frac{R_1}{R_1 + 2R_3}\left(\frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right) V_{1c}$$
$$+ R_1\left(\frac{1}{\alpha_1}\frac{\delta R_2}{R_2} - \frac{\delta\beta_1}{\beta_1}\right)\frac{I_{1c}}{1+\beta_1} \quad (3.33)$$

Similarly, substitution of Eqs. 3.18 through 3.23 into Eqs. 3.14 and 3.16 shows that the CM and DM input currents may be expressed in the forms stated in Eqs. 3.5 through 3.8, where

$$y_{cc} = \frac{1}{(1+\beta_1)(R_1 + 2R_3)} \quad (3.34)$$

$$y_{dd} = \frac{1}{(1+\beta_1)R_1} \quad (3.35)$$

$$y_{cd} = -\frac{1}{(1+\beta_1)R_1}\left(\frac{R_1}{R_1 + 2R_3}\frac{\delta R_1}{R_1} + \alpha_1\frac{\delta\beta_1}{\beta_1}\right) \quad (3.36)$$

$$y_{dc} = -\frac{1}{(1+\beta_1)(R_1 + 2R_3)}\left(\frac{\delta R_1}{R_1} + \alpha_1\frac{\delta\beta_1}{\beta_1}\right) \quad (3.37)$$

$$I_{ce} = y_{cc}E_2 \quad (3.38)$$

$$I_{de} = y_{dc}E_2 \quad (3.39)$$

$$I_{ci} = \frac{1}{1+\beta_1}\left[-\frac{1}{R_1 + 2R_3}V_{1c} - I_{1c}\right.$$
$$\left. + \frac{1}{R_1}\left(\frac{R_1}{R_1 + 2R_3}\frac{\delta R_1}{R_1} + \alpha_1\frac{\delta\beta_1}{\beta_1}\right)V_{1d} + \alpha_1\frac{\delta\beta_1}{\beta_1}I_{1d}\right] \quad (3.40)$$

$$I_{di} = \frac{1}{1+\beta_1}\left[-\frac{1}{R_1}V_{1d} - I_{1d} + \frac{1}{R_1 + 2R_3}\left(\frac{\delta R_1}{R_1} + \alpha_1\frac{\delta\beta_1}{\beta_1}\right)V_{1c}\right.$$
$$\left. + \alpha_1\frac{\delta\beta_1}{\beta_1}I_{1c}\right] \quad (3.41)$$

The comments made in the previous chapter on which quantities are first-order and which are second-order may be verified by examination of Eqs.

3.24 through 3.41. Further insight into the meaning of these results may be obtained by eliminating all unbalances, after which the simplified results can be related to the original circuit by inspection.

To examine the importance of the various effects due to unbalances, we shall examine a typical set of numerical values for the circuit of Fig. 8. If $E_2 = 20$ v, choice of $R_3 = 10$ k will establish the operating collector current of each transistor at about 1 ma, if R_1 is small. If $E_1 = 20$ v, choice of $R_2 = 10$ k will set the quiescent output voltage at about +10 v. These numbers, and arbitrarily chosen values of R_1 and R_4, are shown in Table II. Only 1% unbalances in the homologous elements R_1 and R_2 are assumed, since precision resistors will be normally used.

Numerical values shown in Table II for the transistor parameters represent typical figures appropriate for a wide variety of types. The base-emitter voltage lies in the range 0.1 v to 0.5 v, depending on the operating current level, and is higher for silicon than for germanium transistors at the same current. The saturation current varies widely among different units: The value 0.25 ma shown in Table II is appropriate for a germanium transistor at room temperature, and would be three or four orders of magnitude lower for a silicon unit. The higher value is chosen for the numerical example in order to emphasize its effect. It should be noted that $I_{1c} = 0.25$ ma is the open-base saturation current, and corresponds to an open-emitter saturation current I_{c0} of $I_{1c}/(1 + \beta_1) \approx 5\ \mu$a for the assumed value of common-emitter current gain $\beta_1 = 50$. Unbalances of 10% in the transistor parameters are assumed, although it is likely that some selection from random units would be required to achieve such close matching.

Since in a d-c amplifier temperature drifts will be of interest, it is necessary to know the temperature dependence of the various parameters chosen for the circuit of Fig. 8. It will be assumed that low-temperature-coefficient wire-wound resistors are employed, so that the resistance values may be taken as independent of temperature. On the other hand, all the transistor parameters have significant temperature dependence, and the saturation current and the base-emitter voltage exhibit dependences which are predictable on well-established theoretical grounds.[17–19] The saturation current increases rapidly with rising temperature, at a rate varying from about 20%/°C at low temperatures to about 8%/°C at high temperatures. At any given temperature the rate is 4 to 6%/°C higher for silicon than for germanium units. In Table II, a temperature dependence of 10%/°C is assumed for the saturation current I_{1c}, with an unbalance between the two transistors of 10%.

The base-emitter voltage (at constant collector current) decreases almost linearly with rising temperature, at a rate which varies from about −1.5 mv/°C to about −3 mv/°C. The rate is weakly dependent on the oper-

ating collector current level, being smaller at higher currents. At a given current there is little difference between the rates observed for germanium and for silicon units. In Table II, a temperature dependence of -2.0 mv/°C is assumed for the base-emitter voltage V_{1e}, with an unbalance between the two units of 10%.

The temperature dependence of the common-emitter current gain β is not predictable on theoretical grounds. However, experimental observations[19] indicate that β increases approximately exponentially with rising temperature, at a rate that varies from about 0.2 to 0.5%/°C for either germanium or silicon units. This rate of increase is seen to be considerably less than that of the saturation current. In circuit analysis it is somewhat less easy to account for variations in element values, such as the transistor current gain, than for variations in independent-generator values, such as the saturation current and the base-emitter voltage. Consequently, the temperature variation of β will be neglected in the present numerical example to avoid obscuring illustration of the basic principles of the analysis method. Nevertheless, in a practical case the temperature dependence of β may be important,[13,14] and a means of accounting for it has been given elsewhere.[14] This effect is reconsidered in Section 5.5.

The numerical values shown in Table II are seen to be representative of typical transistors, and to lead to suitable operating points in the single-stage d-c differential amplifier of Fig. 8. The directions of the assumed unbalances are chosen to give worst-case results, that is, to minimize fortuitous cancellation of the effects of unbalances. We may now proceed to substitute these numbers into Eqs. 3.24 through 3.33 to obtain the following results for the amplifier output performance. In some cases intermediate steps are presented in order to illustrate the relative magnitudes of component terms. The order of the terms corresponds to those in the initial equations, and units are volts, milliamperes, and kilohms (or millimhos) unless otherwise stated.

$$A_{cc} = 0.488 \tag{3.42}$$

$$A_{dd} = 81.5 \tag{3.43}$$

$$\frac{1}{H_d} = +0.01 + 201(0.0085 + 0.002)$$

$$= 0.01 + 2.11$$

$$= \frac{1}{0.47} \tag{3.44}$$

$$\frac{1}{H_2} = \frac{1}{H_c} = \frac{1}{201}(0.01 + 0.01 + 0.002)$$

$$= \frac{1}{9.15 \times 10^3} \tag{3.45}$$

$$\frac{1}{A_1} = -\frac{1}{0.487} \tag{3.46}$$

$$\frac{1}{A_2} = 1 \tag{3.47}$$

$$V_{ci} = -V_{1c} + 0.4I_{1c} - 2.12V_{1d} - 0.036I_{1d}$$

$$= 10^{-3}(-300 + 100 - 63.6 - 0.9)$$

$$= -0.264 \tag{3.48}$$

$$\frac{\partial V_{ci}}{\partial T} = 10^{-3}(+2 + 10 + 0.42 - 0.09)$$

$$= +12.4 \text{ mv/°C} \tag{3.49}$$

$$V_{di} = -V_{1d} + 0.002I_{1d} - 0.00011V_{1c} - 0.000176I_{1c}$$

$$= 10^{-3}(-30 + 0.05 - 0.033 - 0.044)$$

$$= -30 \text{ mv} \tag{3.50}$$

$$\frac{\partial V_{di}}{\partial T} = 10^{-3}(+0.2 + 0.0050 + 0.00022 - 0.044)$$

$$= 0.2 \text{ mv/°C} \tag{3.51}$$

The CM gain A_{cc} is considerably less than the DM gain A_{dd} because of the degeneration caused by R_3. The discrimination factor is $F = A_{dd}/A_{cc} = 167$. The DM rejection factor H_d, a second-order quantity, is quite small and is dominated by δR_2. The output fractional unbalance is nevertheless small, because F is large and is given by $U = 1/FH_d = 0.0127$, or 1.27%.

Both the CM rejection factor and the PS_2 rejection factor are large and are dominated by δR_1 and δR_2. It is usual to express rejection factors in decibels; therefore $H_c|_{db} = H_2|_{db} = 20 \log (9.15 \times 10^3) = 79.2$ db. These important quantities mean that, say, a 1-v change in the CM input voltage v_c or a 1-v change in the supply voltage E_2 would give rise to an amplifier DM output voltage indistinguishable from what would result from

a DM input voltage $v_d = 1/9150 \approx 0.11$ mv. This error signal arises from only 1% unbalances in R_1 and R_2 and shows the importance of maintaining balance as closely as possible.

The CM equivalent input voltage V_{ci} produced by the internal sources is dominated by the first-order terms in V_{1c} and I_{1c}, although there is a significant contribution from the second-order term in V_{1d}. The rate of change of this voltage with temperature, the temperature drift $\partial V_{ci}/\partial T$, is dominated by $\partial I_{1c}/\partial T$. The DM equivalent input voltage V_{di} produced by the internal sources and its temperature drift $\partial V_{di}/\partial T$ are dominated by the unbalances V_{1d} and $\partial V_{1d}/\partial T$ in the transistor base-emitter voltages.

It is interesting to calculate the total equivalent input voltages in order to obtain the output offsets in the absence of signals. Each of these is obtained as the sum of the equivalent input voltages arising from the internal and external sources, where the contributions from the external sources are given by Eqs. 3.3 and 3.4:

$$\begin{aligned} V_c &= -0.264 - \frac{1}{0.487} 20 + 20 \\ &= -0.264 - 21 \\ &= -21.26 \end{aligned} \tag{3.52}$$

$$\begin{aligned} V_d &= -0.03 + \frac{1}{9150} 20 \\ &= -0.03 + 0.00218 \\ &= -27.8 \text{ mv} \end{aligned} \tag{3.53}$$

The corresponding output voltages, for v_c and v_d equal to zero, are $-A_{cc}V_c = (-0.488) \times (-21.26) = 10.4$ v and $-A_{dd}V_d = (-81.5) \times (-0.0278) = 2.1$ v. The result for the CM quiescent output voltage checks with the approximate value obtained by inspection from Fig. 8.

Substitution of the typical set of numerical values into Eqs. 3.34 through 3.41 leads to the following results for conditions at the amplifier input:

$$y_{cc} = \frac{1}{1025} \tag{3.54}$$

$$y_{dd} = \frac{1}{5.1} \tag{3.55}$$

$$\begin{aligned} y_{cd} &= -\frac{1}{51}(-0.0005 + 0.98) \\ &= -\frac{1}{52} \end{aligned} \tag{3.56}$$

$$\begin{aligned} y_{dc} &= -\frac{1}{1025}(-0.01 + 0.098) \\ &= -\frac{1}{11{,}650} \end{aligned} \tag{3.57}$$

$$I_{ce} = +19.5\ \mu\text{a} \tag{3.58}$$

$$I_{de} = -1.72\ \mu\text{a} \tag{3.59}$$

$$\begin{aligned} I_{ci} &= -0.000975 V_{1c} - 0.0196 I_{1c} + 0.0192 V_{1d} + 0.00192 I_{1d} \\ &= 10^{-3}(-0.292 - 4.9 + 0.576 + 0.048) \\ &= -4.57\ \mu\text{a} \end{aligned} \tag{3.60}$$

$$\begin{aligned} \frac{\partial I_{ci}}{\partial T} &= 10^{-3}(0.00195 - 0.49 - 0.00384 + 0.0048) \\ &= -0.49\ \mu\text{a}/^\circ\text{C} \end{aligned} \tag{3.61}$$

$$\begin{aligned} I_{di} &= -0.196 V_{1d} - 0.0196 I_{1d} + 0.00000857 V_{1c} + 0.00192 I_{1c} \\ &= 10^{-3}(-5.88 - 0.49 + 0.00257 + 0.48) \\ &= -5.89\ \mu\text{a} \end{aligned} \tag{3.62}$$

$$\begin{aligned} \frac{\partial I_{di}}{\partial T} &= 10^{-3}(0.0392 - 0.049 - 0.00001714 + 0.048) \\ &= 0.038\ \mu\text{a}/^\circ\text{C} \end{aligned} \tag{3.63}$$

We see that both transfer admittances, y_{cd} and y_{dc}, are dominated by $\delta\beta_1$. The CM input current produced by the internal sources, I_{ci}, is dominated by I_{1c}, but the terms in V_{1c} and V_{1d} are significant, and in fact the second-order term in V_{1d} is more important than the first-order term in V_{1c}. The temperature drift of this current is dominated by $\partial I_{1c}/\partial T$. The DM input current produced by the internal sources, I_{di}, is dominated by V_{1d}, although the terms in I_{1d} and I_{1c} could be significant for different numerical values. Its temperature drift is dominated by $\partial V_{1d}/\partial T$, but again different numerical values could make the contributions of $\partial I_{1d}/\partial T$ and $\partial I_{1c}/\partial T$ of considerably greater importance.

The total input currents caused by both internal and external sources, in the absence of input signals, are given by substitution of the last results in Eqs. 3.7 and 3.8:

$$I_c = -4.57 + 19.5$$
$$= +14.9 \ \mu a \tag{3.64}$$
$$I_d = -5.89 - 1.72$$
$$= -7.61 \ \mu a \tag{3.65}$$

Note that I_c is dominated by the external sources, whereas I_d is dominated by the internal sources.

An interesting practical point arises from the presence of four, rather than two, parameters that describe the amplifier input admittances. It is common to determine a "differential input admittance" by measuring the ratio of the input current to the input voltage at one input, with the other input grounded. The admittance so obtained is *not* y_{dd} as we have defined it, however, and it may moreover be different for the two inputs if circuit unbalances are present. The reason for the apparent discrepancy is that a voltage applied to one input alone actually constitutes both CM and DM input signals, and consequently both CM and DM input currents will result. It is easily shown that the two admittances measured as described are given by $[(y_{cc} + y_{dd}) \pm (y_{cd} + y_{dc})]/2$. For the figures given in Eqs. 3.54 through 3.57 the two admittances are 1/11.3 and 1/9.3 millimhos, and differ by about 20% because of the unbalance in β_1.

Analysis of the performance of the amplifier when fed from a zero-impedance signal source is now complete. We turn next to consideration of the effects of finite source impedances.

3.3 Analysis of Effects Produced by Signal Source Impedance

A differential amplifier whose two floating input terminals are at least partially isolated from ground is necessarily driven from a three-terminal signal source, even if the source impedance to ground is only stray coupling. A basic form of signal source with finite impedances is shown in Fig. 12, in which arbitrary source voltages v_{0a} and v_{0b} appear in series with resistances R_a and R_b, and a resistance R_c is common to both branches.

To express the performance of the signal source similarly to the form for the differential amplifier, the network of Fig. 12 may be represented by four open-circuit impedances and two voltage generators which relate, not the actual terminal voltages and currents v_a', v_b', i_a', i_b', but their CM and DM components defined by

$$v_c' = \frac{v_a' + v_b'}{2} \tag{3.66}$$

$$v_d' = \frac{v_a' - v_b'}{2} \tag{3.67}$$

$$i_c' = \frac{i_a' + i_b'}{2} \tag{3.68}$$

$$i_d' = \frac{i_a' - i_b'}{2} \tag{3.69}$$

The required functional relationships are

$$v_c' = z_{cc}i_c' + z_{cd}i_d' + v_{0c} \tag{3.70}$$

$$v_d' = z_{dc}i_c' + z_{dd}i_d' + v_{0d} \tag{3.71}$$

in which z_{cc} and z_{dd} are defined as the CM and DM source impedances, z_{cd} and z_{dc} are the DM-to-CM and the CM-to-DM source transfer impedances, and v_{0c} and v_{0d} are the CM and DM source voltages. If the source is balanced, the transfer impedances z_{cd} and z_{dc} are zero. The properties of any source configuration, for example a bridge network, can be expressed in this general form.

For the specific source configuration of Fig. 12, straightforward analysis leads to the following expressions for the source parameters in Eqs. 3.70 and 3.71:

$$z_{cc} = R + 2R_c \tag{3.72}$$

$$z_{dd} = R \tag{3.73}$$

$$z_{cd} = \delta R \tag{3.74}$$

$$z_{dc} = \delta R \tag{3.75}$$

$$v_{0c} = \frac{v_{0a} + v_{0b}}{2} \tag{3.76}$$

$$v_{0d} = \frac{v_{0a} - v_{0b}}{2} \tag{3.77}$$

where

$$R \equiv \frac{R_a + R_b}{2} \tag{3.78}$$

$$\delta R \equiv \frac{R_a - R_b}{2} \tag{3.79}$$

For purposes of illustration, a typical set of values is assumed for the source resistances and displayed in Table III. The four source open-circuit impedances are then

$$z_{cc} = 10.5 \text{ k} \tag{3.80}$$

$$z_{dd} = 0.5 \text{ k} \tag{3.81}$$

$$z_{cd} = z_{dc} = -0.05 \text{ k} \tag{3.82}$$

The next step is to obtain expressions for the amplifier CM and DM input voltages v_c and v_d in terms of the source CM and DM voltages v_{0c} and v_{0d}. Since the source is loaded by the amplifier input, this process involves solution of the circuit in Fig. 13. Algebraically, the required result is obtained by simultaneous solution of Eqs. 3.5, 3.6, 3.70, and 3.71, with $i_c = -i_c'$, $i_d = -i_d'$, $v_c = v_c'$, and $v_d = v_d'$. The results are

$$v_c = \frac{1}{1 + z_{cc}y_{cc}} v_{0c} - \frac{z_{cc}y_{cd} + z_{cd}y_{dd}}{(1 + z_{cc}y_{cc})(1 + z_{dd}y_{dd})} v_{0d} - \frac{z_{cc}}{1 + z_{cc}y_{cc}} I_c - \frac{z_{cd} - z_{cc}z_{dd}y_{cd}}{(1 + z_{cc}y_{cc})(1 + z_{dd}y_{dd})} I_d \tag{3.83}$$

$$v_d = -\frac{z_{dc}y_{cc} + z_{dd}y_{dc}}{(1 + z_{dd}y_{dd})(1 + z_{cc}y_{cc})} v_{0c} + \frac{1}{1 + z_{dd}y_{dd}} v_{0d} - \frac{z_{dd}}{1 + z_{dd}y_{dd}} I_d - \frac{z_{dc} - z_{cc}z_{dd}y_{dc}}{(1 + z_{dd}y_{dd})(1 + z_{cc}y_{cc})} I_c \tag{3.84}$$

Certain good approximations, based on the assumption that the percentage unbalances in the source and in the amplifier are small, have been made in the derivation of these two equations.

It is convenient to express these results in forms analogous to those used to represent the amplifier output performance. Thus

$$v_c = A_{ccs}\left(v_{0c} + V_{cs} + \frac{1}{H_{ds}} v_{0d}\right) \tag{3.85}$$

$$v_d = A_{dds}\left(v_{0d} + V_{ds} + \frac{1}{H_{cs}} v_{0c}\right) \tag{3.86}$$

in which

$$V_{cs} \equiv V_{cis} + \frac{1}{A_{1s}} E_1 + \frac{1}{A_{2s}} E_2 \tag{3.87}$$

$$V_{ds} \equiv V_{dis} + \frac{1}{H_{1s}} E_1 + \frac{1}{H_{2s}} E_2 \tag{3.88}$$

The various source performance parameters used in these expressions to relate the amplifier input voltages to the source voltages are analogous to those previously employed to relate the amplifier output to input voltages, and therefore corresponding symbols are used, but with an added subscript s.

By comparison of Eqs. 3.85 through 3.88 with Eqs. 3.83 and 3.84, and with use of Eqs. 3.7, 3.8, 3.38, and 3.39, the following expressions are obtained for the source performance parameters.

$$A_{ccs} \equiv \frac{1}{1 + z_{cc}y_{cc}} \tag{3.89}$$

$$A_{dds} \equiv \frac{1}{1 + z_{dd}y_{dd}} \tag{3.90}$$

$$\frac{1}{H_{ds}} \equiv -\frac{z_{cc}y_{cd} + z_{cd}y_{dd}}{1 + z_{dd}y_{dd}} \tag{3.91}$$

$$\frac{1}{H_{cs}} \equiv -\frac{z_{dc}y_{cc} + z_{dd}y_{dc}}{1 + z_{cc}y_{cc}} \tag{3.92}$$

$$\frac{1}{A_{1s}} = 0 \tag{3.93}$$

$$\frac{1}{A_{2s}} = -z_{cc}y_{cc} \tag{3.94}$$

$$\frac{1}{H_{1s}} = 0 \tag{3.95}$$

$$\frac{1}{H_{2s}} = \frac{1}{H_{cs}} \tag{3.96}$$

$$V_{cis} \equiv -z_{cc}I_{ci} - \frac{z_{cd} - z_{cc}z_{dd}y_{cd}}{1 + z_{dd}y_{dd}} I_{di} \tag{3.97}$$

$$V_{dis} \equiv -z_{dd}I_{di} - \frac{z_{dc} - z_{cc}z_{dd}y_{dc}}{1 + z_{cc}y_{cc}} I_{ci} \tag{3.98}$$

Substitution of the typical sets of numerical values previously used leads to the following results:

$$A_{ccs} \approx 1 \tag{3.99}$$

$$A_{dds} = 0.912 \tag{3.100}$$

$$\frac{1}{H_{ds}} = +\frac{0.202 + 0.0098}{1.098} = +\frac{1}{5.18} \tag{3.101}$$

$$\frac{1}{H_{2s}} = \frac{1}{H_{cs}} = +10^{-5}(4.88 + 4.29)$$

$$= \frac{1}{1.09 \times 10^4} \tag{3.102}$$

$$\frac{1}{A_{2s}} = -\frac{1}{97.5} \tag{3.103}$$

$$V_{cis} = -10.5I_{ci} - 0.0465I_{di}$$

$$= (+49.5 + 0.279)\ \text{mv}$$

$$= +49.8\ \text{mv} \tag{3.104}$$

$$\frac{\partial V_{cis}}{\partial T} = (+5.25 - 0.0016)\ \text{mv/°C}$$

$$= +5.25\ \text{mv/°C} \tag{3.105}$$

$$V_{dis} = -0.5I_{di} + 0.05I_{ci} \tag{3.106a}$$

$$= (3 - 0.236)\ \text{mv} \tag{3.106b}$$

$$= +2.76\ \text{mv} \tag{3.106}$$

$$\frac{\partial V_{dis}}{\partial T} = (-0.02 - 0.025)\ \text{mv/°C}$$

$$= -0.045\ \text{mv/°C} \tag{3.107}$$

The CM and DM source "gains" A_{ccs} and A_{dds}, of course, are actually losses arising from the potential divider actions of the source and amplifier impedances. The source DM rejection factor H_{ds} is dominated by the term in y_{cd}, but both the term in y_{dc} and the term in z_{dc} are of comparable magnitude in the expression for the CM rejection factor H_{cs}. The CM equivalent source voltage V_{cis} arising from the amplifier internal sources, and also its temperature drift, are dominated by the I_{ci} term. The DM equivalent source voltage V_{dis} is dominated by the I_{di} term, but the I_{ci} term is significant. Both terms are of comparable importance in determining the temperature drift of this voltage.

3.4 Performance Parameters for the Combined Single-Stage Amplifier and Source

The performance parameters for the combined single-stage amplifier and source are the quantities which relate the amplifier CM and DM output

voltages v_{1c} and v_{1d} to the CM and DM source voltages v_{0c} and v_{0d}. The relationships may be expressed as

$$v_{1c} = -A_{cct}\left(v_{0c} + V_{ct} + \frac{1}{H_{dt}} v_{0d}\right) \tag{3.108}$$

$$v_{1d} = -A_{ddt}\left(v_{0d} + V_{dt} + \frac{1}{H_{ct}} v_{0c}\right) \tag{3.109}$$

in which

$$V_{ct} \equiv V_{cit} + \frac{1}{A_{1t}} E_1 + \frac{1}{A_{2t}} E_2 \tag{3.110}$$

$$V_{dt} \equiv V_{dit} + \frac{1}{H_{1t}} E_1 + \frac{1}{H_{2t}} E_2 \tag{3.111}$$

The various performance parameters used in these expressions again correspond to those used for the amplifier and source separately, and corresponding symbols are used with an added subscript t to indicate "total" values.

The total performance parameters may be evaluated by elimination of v_c and v_d from Eqs. 3.1, 3.2, and 3.85 through 3.88. With certain good approximations, based on the assumption that the unbalances are small, the results are:

$$A_{cct} \equiv A_{ccs}A_{cc} \tag{3.112}$$

$$A_{ddt} \equiv A_{dds}A_{dd} \tag{3.113}$$

$$\frac{1}{H_{dt}} \equiv \frac{1}{H_{ds}} + \frac{A_{dds}}{A_{ccs}H_d} \tag{3.114}$$

$$\frac{1}{H_{ct}} \equiv \frac{1}{H_{cs}} + \frac{A_{ccs}}{A_{dds}H_c} \tag{3.115}$$

$$\frac{1}{A_{1t}} \equiv \frac{1}{A_{1s}} + \frac{1}{A_{ccs}A_1} \tag{3.116}$$

$$\frac{1}{A_{2t}} \equiv \frac{1}{A_{2s}} + \frac{1}{A_{ccs}A_2} \tag{3.117}$$

$$\frac{1}{H_{1t}} \equiv \frac{1}{H_{1s}} + \frac{1}{A_{dds}H_1} \tag{3.118}$$

$$\frac{1}{H_{2t}} \equiv \frac{1}{H_{2s}} + \frac{1}{A_{dds}H_2} \tag{3.119}$$

$$V_{cit} \equiv V_{cis} + \frac{1}{A_{ccs}} V_{ci} \tag{3.120}$$

$$V_{dit} \equiv V_{dis} + \frac{1}{A_{dds}} V_{di} \tag{3.121}$$

It is seen that effects due to the source and to the amplifier are approximately additive, since the source "gains" A_{ccs} and A_{dds} are close to unity.

Substitution of values from Eqs. 3.42 through 3.51 and 3.99 through 3.107 leads to the following complete numerical results for the amplifier circuit of Fig. 8:

$$A_{cct} = 0.488 \tag{3.122}$$

$$A_{ddt} = 74.4 \tag{3.123}$$

$$\frac{1}{H_{dt}} = \frac{1}{5.18} + \frac{1}{0.457}$$

$$= \frac{1}{0.421} \tag{3.124}$$

$$\frac{1}{H_{2t}} = \frac{1}{H_{ct}} = \frac{1}{10^3}\left(\frac{1}{10.88} + \frac{1}{8.35}\right)$$

$$= \frac{1}{4.71 \times 10^3} \tag{3.125}$$

$$\frac{1}{A_{1t}} = 0 - \frac{1}{0.487}$$

$$= -\frac{1}{0.487} \tag{3.126}$$

$$\frac{1}{A_{2t}} = -\frac{1}{97.5} + 1$$

$$\approx 1 \tag{3.127}$$

$$\frac{1}{H_{1t}} = 0 + \frac{1}{8.94 \times 10^3}$$

$$= \frac{1}{8.94 \times 10^3} \tag{3.128}$$

$$V_{cit} = (49.8 - 264) \text{ mv}$$

$$= -214 \text{ mv} \tag{3.129}$$

$$\frac{\partial V_{cit}}{\partial T} = (+5.25 + 13.6) \text{ mv/°C}$$

$$= +18.9 \text{ mv/°C} \tag{3.130}$$

$$V_{dit} = (2.76 - 33.2) \text{ mv}$$

$$= -30.4 \text{ mv} \tag{3.131}$$

$$\frac{\partial V_{dit}}{\partial T} = (-0.045 + 0.219) \text{ mv/°C}$$

$$= +0.174 \text{ mv/°C} \tag{3.132}$$

Numerical expressions for the complete CM and DM output voltages may be obtained by substitution of numbers directly into Eqs. 3.108 and 3.109:

$$v_{1c} = -0.488\left(v_{0c} - 21.21 + \frac{1}{0.421} v_{0d}\right) \tag{3.133}$$

$$v_{1d} = -74.4\left(v_{0d} - 0.0239 + \frac{1}{4710} v_{0c}\right) \tag{3.134}$$

The quiescent output voltages, for v_{0c} and v_{0d} equal to zero, are thus $(-0.488) \times (-21.21) = 10.38$ v and $(-74.4) \times (-0.0239) = 1.78$ v.

The total CM gain A_{cct} is essentially the same as A_{cc}, that of the amplifier alone, and the total DM gain A_{ddt} is only slightly less than A_{dd}. The total discrimination factor is $F_t = A_{ddt}/A_{cct} = 152$. The total DM rejection factor is slightly poorer than that for the amplifier alone, and the output fractional unbalance increases from 1.27% to $U_t = 1/F_tH_{dt} = 0.0156$, or 1.56%.

The total CM rejection factor H_{ct}, equal to the PS_2 rejection factor H_{2t}, is reduced by almost a factor of 2 from the value for the amplifier alone, from 79.2 db to 73.5 db. It is worth noting that this reduction is due almost equally to the combination of the average DM source resistance z_{dd} with the amplifier CM-to-DM input transfer admittance y_{dc} and to the combination of the unbalance DM source resistance z_{cd} with the amplifier CM input admittance y_{cc}.

The total CM equivalent input voltage V_{cit}, and its temperature drift, are dominated by the amplifier alone, but there is a significant contribution from the CM input current I_{ci} flowing in the CM source resistance z_{cc}. The total DM equivalent input voltage V_{dit}, and its temperature drift, are dominated by the amplifier alone, with only a small contribution produced by the DM input current I_{di} flowing in the DM source resistance z_{dd}.

It is of interest to examine the dependence of the results on the various circuit parameters. A small value of R_1 gives large DM gain and ensures

that the contributions of I_{1d}, V_{1e}, and I_{1e} to the DM equivalent input drift are small (Eqs. 3.33 and 3.50). In addition, the value of H_e will be large (Eq. 3.27). On the other hand, the amplifier input admittances y_{dd} and y_{ed} will be large (Eqs. 3.35 and 3.36), which will increase the input potential divider action (reduce A_{dds}) (Eq. 3.90) and will degrade H_{ds} (Eq. 3.91). However, this last effect is not serious because the contribution of H_{ds} to H_{dt} is small (Eqs. 3.114 and 3.124). In general, then, it is desirable to make R_1 small. Unbalance in R_1 seriously affects the total CM rejection factor H_{et}, both through H_e (Eqs. 3.27 and 3.45) and through H_{es} (Eqs. 3.37, 3.57, and 3.92).

Increase in R_2 increases both the CM and DM gains, but the DM gain increases more slowly because of the shunting effect of R_4 (Eqs. 3.24 and 3.25). Hence the discrimination factor and the output fractional unbalance are degraded if R_2 approaches $R_4/2$. Unbalance in R_2 seriously affects H_{et} through H_e (Eqs. 3.27 and 3.45) and significantly affects H_d (Eqs. 3.26 and 3.44).

A large value of β_1 is beneficial in all respects since all four amplifier input admittances are reduced (Eqs. 3.34 through 3.37), and all components of the amplifier input currents are reduced (Eqs. 3.38 through 3.41). Unbalance in β_1 primarily affects H_{et} through H_{es} and y_{de} (Eqs. 3.37, 3.57, 3.91, and 3.102), but the contribution through H_e is negligible unless the unbalances in R_1 and R_2 are very small (Eqs. 3.27 and 3.45).

Both the CM and the DM source impedances z_{ee} and z_{dd} should be kept low, not only to reduce the potential divider action at the amplifier input, but also to minimize the equivalent input voltages developed by the amplifier input currents (Eqs. 3.97 and 3.98). Another important disadvantage of large values of z_{ee} and z_{dd} is the serious effect on the rejection factors H_{ds} and H_{es} (Eqs. 3.91, 3.92, 3.101, and 3.102). Any unbalance in the source impedance also seriously affects H_{es}.

The total CM equivalent input voltage V_{eit} may have significant contributions from several sources. For the assumed set of numbers, the contributions of the first-order terms V_{1e} and I_{1e} are the most important. The effect produced by V_{1e} is felt primarily through the component V_{ei} (Eqs. 3.32 and 3.48), but that arising from I_{1e} is felt both through V_{ei} (Eqs. 3.32 and 3.48) and through V_{eis} (Eqs. 3.40, 3.60, and 3.104). However, the total CM equivalent input voltage is not of great interest because of the low CM gain.

The total DM equivalent input voltage V_{dit} is a quantity of major importance. For the chosen numbers, this quantity is dominated by the unbalance V_{1d} in the transistor base-emitter voltages (Eqs. 3.33, 3.50, 3.121, and 3.131). If the transistors are selected for closer matching, or if some method of compensation is employed, this dominant term can be greatly

reduced, thus exposing the next largest term, which is that produced by the unbalance in the transistor saturation currents I_{di} flowing in the DM source resistance z_{dd} (Eqs. 3.98, 3.106, and 3.131). Alternatively, a larger DM source resistance could make this term significant even in the presence of appreciable unbalance in the base-emitter voltages. On the other hand, use of silicon transistors would essentially eliminate any contribution from the saturation currents. Similar remarks apply to the temperature drift of the total DM equivalent input voltage, which again is dominated by the term in the unbalance base-emitter voltage unless much more careful matching or compensation is employed.

It is of particular interest to examine the effect of the value of R_3 on the various performance quantities. It may be seen that the largest possible value of R_3 is in all respects beneficial. First-order DM quantities are unchanged, but first-order CM quantities are improved: the CM gain A_{ee} is reduced (Eqs. 3.24 and 3.112), and hence the discrimination factor F_t is increased; the CM and CM-to-DM input admittances y_{ee} and y_{de} are reduced (Eqs. 3.34 and 3.37), which increases the CM and PS_2 rejection factors H_{et} and H_{2t} (Eqs. 3.27 and 3.92); and the contribution of the second-order term in V_{1e} to the DM equivalent input drift, although already negligible, is reduced even further (Eq. 3.33). The occurrence of R_3 in the numerator of certain expressions for CM quantities (Eqs. 3.26, 3.28, and 3.32) does not detract from the desirability of a large value for R_3 because, as far as the amplifier output is concerned, these quantities are always multiplied by A_{eet}, which contains R_3 in its denominator.

The equations of this chapter describe the various contributions to the CM and to the DM output voltages separately. It is implicitly assumed that only the DM output voltage is of practical interest; however, the DM output is "floating" with respect to ground and requires a floating load. Although this is no problem if the load is, say, a recorder, it is sometimes necessary to provide a grounded output from the amplifier. There is a great temptation in such applications to connect the load between one output and ground. However, this procedure is fraught with danger: The voltage across the load is then the sum of the CM and DM output voltages v_{1e} and v_{1d}, and even though the output voltage may appear to be little different from v_{1d} alone, it actually contains contributions from several other sources. As can be seen by adding Eqs. 3.108 and 3.109, the DM equivalent input drift would contain components produced by V_{et} as well as by V_{dt}, and the CM rejection factor would be dependent on H_{dt} as well as H_{et}. The problem of attaining satisfactory performance is much more difficult with a grounded output, and merely making the CM gain vanishingly small is not sufficient. The terms A_{eet}/H_{dt} must also be minimized, and, as can be seen from Eqs. 3.24, 3.26, and 3.32, only making R_3

very large will not accomplish this. In general, more sophisticated methods must be employed[14] to derive a grounded output from a differential amplifier.

We may conclude that in the basic differential-amplifier circuit of Fig. 8, when the first-order DM properties are of prime concern, the performance is enhanced by using as large a value of R_3 as possible. This decreases the CM gain while leaving the DM gain unaffected and reduces the undesirable CM-to-DM interaction effects, and thus leads to improved CM and PS rejections. Nevertheless, offsets and drifts caused by unbalanced independent generators, such as those arising from unequal transistor base-emitter voltages or saturation currents, cannot be improved in this way. The advantages to be gained by large values of R_3 are sufficiently worthwhile that modifications in the basic circuit of Fig. 8 should be introduced to enhance the effect still further. Some of the modifications will be discussed in the following chapters.

Chapter Four

Improvements on the basic differential-amplifier circuit

In the previous chapter it was verified by detailed analysis, using the sequential method, that the effects of unbalances in the basic differential-amplifier circuit of Fig. 8 can be reduced by making the common emitter resistance R_3 very large. There are, however, other considerations in addition to those treated in Chapter 3. Increase in R_3 would reduce the operating currents of the transistors and hence lead to higher internal emitter resistance and lower current gain. This effect can be eliminated by increasing the negative supply voltage E_2, as R_3 is increased, in such a way that the transistor operating currents remain constant. Clearly, the combination of the elements R_3 and E_2 then approaches a constant current sink.

In this chapter a circuit for practical realization of a constant current will be considered, and a further modification involving CM negative feedback will be introduced.

4.1 Use of a Constant-Current Generator

The development of a circuit that allows flexibility in the choice of the emitter coupling resistance of a differential amplifier is shown in Fig. 14. The original voltage source and series resistance of Fig. 8, isolated in Fig. 14*a*, are converted to a Norton equivalent in Fig. 14*b*, and in Fig. 14*c* the resulting current generator is realized by a suitably biased transistor Q_3. The constant-current transistor is the counterpart of the constant-current pentode introduced by Goldberg[4] into the vacuum-tube differential amplifier.

The circuit of Fig. 14*c* provides essentially the same performance as that of Fig. 14*a*, if R_3 is the same in both cases and if circuit values are chosen

so that the Q_3 collector current $I_3 = E_2/R_3$. Straightforward analysis of Fig. 14c, with the assumption that the transistor can be represented solely by a β-current generator $\beta_3 = \alpha_3/(1 - \alpha_3)$, leads to the result $I_3 = E_2/R_3'$, where

$$R_3' \equiv \frac{R_6 + R_7}{\alpha_3 R_7}\left(R_6 + \frac{R_6 \| R_7}{1 + \beta_3}\right) \tag{4.1}$$

For the same I_3, and with the same value of E_2, R_3' must be set equal to R_3.

This procedure only ensures that the operating conditions in the circuit of Fig. 8 remain the same when the circuit of Fig. 14c replaces that of Fig. 14a, as in the complete circuit of Fig. 15. The advantage to be gained by the substitution is that R_3 may now be increased without affecting the operating conditions, and thus all the performance benefits predicted in the previous chapter may be realized. Some limitations should be noted, however.

Although the CM rejection factor H_c (Eq. 3.27) becomes very large as R_3 becomes large, the PS_2 rejection factor H_2 does not increase with H_c, and in fact it remains the same as in the original circuit of Fig. 8. This is because the sensitivity of the operating currents of the transistors in the differential stage to the supply voltage E_2 is unchanged if Eq. 4.1 is satisfied. Equation 3.31 therefore no longer holds if R_3 differs from its original value. Additional terms will be present in Eqs. 3.32 and 3.33 that are produced by the base-emitter voltage and saturation current of the transistor in Fig. 14c, effects that were not included in the simple derivation of Eq. 4.1.

In view of the many benefits of making R_3 very large, it would seem obvious that the ultimate performance could be obtained by making R_3 infinite in the circuit of Fig. 15. The CM gain A_{cc} would then become zero (Eq. 3.24), and the CM rejection factor H_c would become infinite (Eq. 3.27), even in the presence of unbalances in R_1, R_2, and β_1. Unfortunately, it is not possible in practice to achieve this desirable result; this is the most important limitation of the circuit in Fig. 15. A transistor is not an ideal current generator, and there is inevitably a finite impedance between the collector and the base or the emitter. These impedances have been neglected in the present analysis; however, even though the circuit resistor R_3 may be omitted, the element R_3 should be retained in the equivalent circuits as an approximate representation of the finite collector resistances of all three transistors. Unbalances in the collector resistances of Q_{1a} and Q_{1b}, of course, will not be accounted for in this way.

In any case, it is not possible to realize the condition of infinite R_3 in practice, with the result that the CM gain is not quite zero, and the CM rejection factor is not infinite and depends on the circuit unbalances (unless they fortuitously cancel). This limitation has been noted in other contexts,[8,10,11] where analysis of tube differential amplifiers, in which finite

plate resistance is taken into account, shows that the CM rejection factor is related to the unbalance in the tube amplification factors even in the presence of infinite common cathode impedance. This conclusion is verified in Appendix II. Klein[11] has devoted considerable attention to cascode arrangements for the purpose of increasing the effective amplification factor. An analogous conclusion is, of course, obtained for transistor differential amplifiers.

It is possible to increase H_c by various circuit modifications. One method, due to Hilbiber,[20] involves replacement of each transistor in the differential stage by a multiple-stage amplifier whose output resistance is made very high by negative feedback. Nevertheless, the contribution of the finite output resistance of the constant-current transistor to the effective value of R_3 still remains. Another method, to be discussed next, permits H_c to be increased by an arbitrarily large factor through the application of CM negative feedback.

4.2 Advantages of Common-Mode Negative Feedback

We have seen that a large value of the common emitter resistance R_3 in the differential-amplifier circuit of Fig. 15 leads to improved performance, through reduction of the CM gain and increase of the CM rejection factor. Since there is a practical limit to the effective magnitude of R_3 owing to finite transistor collector resistances, it is desirable to develop a modified circuit design which would permit these performance limits to be exceeded without compromising other performance criteria. Since negative feedback reduces gain, this would be a suitable technique to employ. Only the CM, and not the DM, gain is to be reduced, however, and so the feedback should be effective for CM signals alone. The principle of CM negative feedback was introduced in vacuum-tube differential amplifiers by Offner[3]. It will be shown that CM feedback leads not only to this anticipated result, but also to the desired increase in CM rejection factor.

Common-mode negative feedback may in principle be introduced into the circuit of Fig. 15 by including in the voltage E_2 a component related to the CM output voltage v_{1c}. Such a component may be derived,[7] for example, from a center tap on R_4, since the voltage at this point is equal to the CM output voltage v_{1c}, but is at virtual ground with respect to the DM output voltage. A somewhat more elaborate circuit to accomplish this effect is discussed in the next chapter; here, to illustrate the principle, we assume that an unspecified subsidiary circuit exists such that $E_2 = kv_{1c}$. Thus the voltage E_2 is k times the CM output voltage, and is independent of the DM output voltage. The necessary constant power supply component of E_2 is

neglected, since this affects only operating conditions and is irrelevant as far as the signal properties of the amplifier are concerned.

Since the circuits of Figs. 8 and 15 are the same as far as the input signals are concerned, the CM and DM output voltages in the circuit of Fig. 15 are given by

$$v_{1c} = -A_{cc}\left(v_c + V_{ci} + \frac{1}{H_d}v_d + \frac{1}{A_1}E_1 + E_2\right) \tag{4.2}$$

$$v_{1d} = -A_{dd}\left(v_d + V_{di} + \frac{1}{H_c}v_c + \frac{1}{H_1}E_1 + \frac{1}{H_c}E_2\right) \tag{4.3}$$

These equations are obtained from Eqs. 3.1 and 3.2. It is assumed that the operating conditions in the circuits of Figs. 8 and 15 are adjusted to be the same, so that the results $A_2 = 1, H_2 = H_c$ (Eqs. 3.29 and 3.31) may be employed. If CM feedback is now applied according to the relation

$$E_2 = kv_{1c} \tag{4.4}$$

the results become

$$v_{1c} = -\frac{A_{cc}}{1+G_c}\left(v_c + V_{ci} + \frac{1}{H_d}v_d + \frac{1}{A_1}E_1\right) \tag{4.5}$$

$$v_{1d} = -A_{dd}\left[v_d + \left(V_{di} - \frac{1}{H_c}\frac{G_c}{1+G_c}V_{ci}\right) + \frac{1}{(1+G_c)H_c}v_c + \left(\frac{1}{H_1} - \frac{1}{H_cA_1}\frac{G_c}{1+G_c}\right)E_1\right] \tag{4.6}$$

where

$$G_c \equiv kA_{cc} \tag{4.7}$$

is the *CM loop gain.* The feedback is negative if k is positive, since the CM gain is reduced by the feedback factor $(1 + G_c)$. An approximation has been made in the derivation of Eq. 4.6: the coefficient of v_d is actually $[1 - G_c/H_cH_d(1 + G_c)]$, but for $H_c = 9.15 \times 10^3$ (Eq. 3.45) and $H_d = 0.47$ (Eq. 3.44) this quantity differs from unity by less than 0.0233.

Equations 4.5 and 4.6 verify that CM negative feedback reduces the CM gain and that the DM gain is hardly changed; the actual small decrease in the DM gain is a third-order effect. Another important result is that the CM rejection factor becomes $(1 + G_c)H_c$ instead of H_c, and is thus increased by the same factor the CM gain is decreased by. However, this is the CM rejection factor with respect to the signal voltages at the amplifier input terminals, and it has been seen in Chapter 3 that significant degradation in total CM rejection factor can arise from finite signal source impedances. It is therefore necessary to determine the effect of CM feedback on the amplifier input admittances, so that the effects of finite signal source impedances can be examined.

Since the results for the circuit of Fig. 8 apply to the circuit of Fig. 15, the relations between the signal input currents and voltages are

$$i_c = y_{cc}v_c + y_{cd}v_d + I_{ci} + y_{cc}E_2 \tag{4.8}$$

$$i_d = y_{dc}v_c + y_{dd}v_d + I_{di} + y_{dc}E_2 \tag{4.9}$$

These equations are obtained by combining Eqs. 3.5 through 3.8 and incorporating the results of Eqs. 3.38 and 3.39. If CM feedback is applied so that $E_2 = kv_{1c}$, the results become

$$i_c = \frac{y_{cc}}{1+G_c}v_c + \left(y_{cd} - \frac{y_{cc}}{H_d}\frac{G_c}{1+G_c}\right)v_d + \left[I_{ci} - y_{cc}\frac{G_c}{1+G_c}\left(V_{ci} + \frac{1}{A_1}E_1\right)\right] \tag{4.10}$$

$$i_d = \frac{y_{dc}}{1+G_c}v_c + y_{dd}v_d + \left[I_{di} - y_{dc}\frac{G_c}{1+G_c}\left(V_{ci} + \frac{1}{A_1}E_1\right)\right] \tag{4.11}$$

The coefficient of v_d in Eq. 4.11 actually differs from the first-order y_{dd} by a small third-order term; otherwise, no approximations have been made in the derivation of Eqs. 4.10 and 4.11.

From these results it is seen that both the CM and the CM-to-DM input admittances are reduced by the application of CM negative feedback. Since each is reduced by the feedback factor $(1 + G_c)$ it follows from Eq. 3.92 that the CM rejection factor H_{cs} produced by the source is increased by $(1 + G_c)$, and since H_c is also increased by the same factor, the total CM rejection factor H_{ct} is increased by $(1 + G_c)$.

It may be concluded that the application of CM negative feedback affects the first-order DM performance only slightly and has several advantageous effects on certain other performance parameters: the CM gain is reduced, thus the discrimination factor is increased and the output fractional unbalance is reduced; the CM and the CM-to-DM input admittances are reduced; and the total CM rejection factor is increased. The factor by which these various quantities change is in all cases the feedback factor $(1 + G_c)$, and it is therefore desirable to make the CM loop gain G_c as high as possible. A qualitative description of the mechanism of CM feedback has been given by Birt;[12] however, he discusses only its effects on the fractional output unbalance and on the discrimination factor. The effect of CM feedback on CM rejection has been recognized[3,10] but appears not to have been discussed in detail. Its effect on the input admittances seems not to have been noted.

Certain quantities for which the application of CM feedback is invariably beneficial have so far been singled out for discussion. It may be observed that in all these cases the expressions are modified as though R_3 were everywhere multiplied by $(1 + G_c)$. Another point of view is therefore to imagine that CM negative feedback causes the effective value of R_3 to be $(1 + G_c)$ times its actual value, thus permitting the practical limitations on the amplifier performance caused by the unavoidably finite R_3 to be exceeded.

There are effects of CM feedback other than those so far discussed. For example, the DM equivalent input drift term and the PS_1 rejection factor in Eq. 4.6 are modified; so (in Eqs. 4.10 and 4.11) are the DM-to-CM transfer input admittance and the CM and DM input currents produced by both internal and external sources. The nature and magnitude of these modifications are not immediately obvious. In addition, the simple relationship $E_2 = kv_{1c}$ employed in the derivation represents the feedback circuit only in principle, and in the results additional terms that arise from effects in a practical feedback circuit should be present. It is therefore not worthwhile to examine in any detail the remaining modified terms in Eqs. 4.5, 4.6, 4.10, and 4.11; instead, a differential amplifier including a practical CM feedback circuit is treated in the next chapter. It will serve as a further example of the sequential method of solution of unbalanced symmetrical circuits.

Chapter Five

Analysis of a two-stage transistor d-c differential amplifier with common-mode negative feedback

In this chapter the properties of a particular transistor d-c differential amplifier employing common-mode negative feedback will be discussed. The circuit, shown in Fig. 16, is similar to several that have been used in practical vacuum-tube and transistor amplifiers.[12–14]

Before plunging into the analysis of the circuit of Fig. 16, it is essential to understand the principle of operation and to have a qualitative appreciation of the first-order relationships. Only with such an understanding can the algebra involved in the analysis of unbalance effects be kept under control. An expression for the CM loop gain is then derived, after which application of the sequential method for analysis of unbalances is described. The resulting expressions for the performance parameters are compared with those for the circuit of Fig. 8, and the improvements anticipated from the presence of CM feedback are verified. It will be shown, however, that these improvements are obtained at a price, and that CM feedback is not an entirely unmixed blessing; some performance parameters may even be degraded. In particular, for the numerical values assumed, the PS rejection factors are finite and less than the CM rejection factor by more than an order of magnitude.

5.1 Mechanism of the Circuit Operation

The circuit of Fig. 16 represents a two-stage differential amplifier, of which the fitsr stage is similar to that discussed in Chapter 3, and CM negative feedback is applied according to the principle discussed in Chapter 4. The output is now taken from the collectors of the second stage, and

the output of the first stage is applied to the two bases of the second-stage transistors which replace the bridging resistance R_4. The junction of the second-stage emitters corresponds to the center of R_4, from which the CM negative feedback is derived. All voltages are referred to ground. The resistance R_3 is retained since, as described in Chapter 4, it represents the finite collector resistances of the three associated transistors Q_{1a}, Q_{1b}, Q_3 even if no circuit resistor is present. The numerical values of elements that also appear in Fig. 8 are maintained, so that the performance of the two circuits may be meaningfully compared. Should any unbalance be present, the numerical values for homologous elements represent the average of the pair.

It is not immediately apparent whether the operating currents of the first-stage transistors will be the same as in Fig. 8. To investigate this, consider the principle of operation of the CM feedback loop. From an arbitrary point, say the collector of Q_3, the path through the CM feedback loop passes through Q_{1a} and Q_{1b} (in parallel), through Q_{2a} and Q_{2b} (in parallel), through the potential divider R_6 and R_7, and thence to the base of Q_3. This path resembles that through a series voltage regulator circuit, in which $(E_3 - E_2)$ corresponds to the reference voltage and $(v_x + E_3)$ corresponds to the regulated output voltage. Thus Q_3 represents the error amplifier. The loop gain of this voltage regulator circuit is identical with the CM loop gain G_c introduced in Chapter 4, and it is desired to be large. If this is so, the voltage $(v_x + E_3)$ will be determined principally by the potential divider ratio and by the reference voltage $(E_3 - E_2)$; there will be little voltage drop across R_5, and the value of this element is then immaterial as long as it is small enough. However, since the internal base-emitter voltage V_{3e} of Q_3 is effectively in series with the reference voltage, its value is not negligible and should be included, so that the approximate equation determining v_x is

$$\frac{R_7}{R_6 + R_7}(v_x + E_3) = E_3 - E_2 + V_{3e} \qquad (5.1)$$

For the numerical values shown in Fig. 16, and for $V_{3e} = 0.3$ v, Eq. 5.1 gives $v_x = 10.9$ v. The operating current of Q_{1a}, or Q_{1b}, may then be found approximately as follows. If the internal base-emitter voltage V_{2e} of Q_2 is 0.3 v, the voltage v_{1c} at the collector of Q_1 is $(v_x + V_{2e}) = 11.2$ v. The current in R_2 is therefore $[E_1 - (v_x + V_{2e})]/R_2 = 0.88$ ma. If the base current of Q_2 is neglected, the collector current of Q_{1a}, or Q_{1b}, is also approximately 0.88 ma. The corresponding current in the circuit of Fig. 8, in the absence of an input signal, may be found from Fig. 9*a* as approximately $\alpha_1(E_2 - V_{1e})/(R_1 + 2R_3) = 0.96$ ma. The operating conditions of the first stage in Fig. 8 and in Fig. 16, although not identical, are therefore sufficiently similar that the performance of the two circuits may be compared.

Some additional useful information may be deduced from the preceding approximate calculations. First, it may be noted that the collector current of Q_{1a} or Q_{1b} is determined by completely different mechanisms in the circuits of Figs. 8 and 16: In Fig. 8, it is determined essentially by E_2 and R_3; in Fig. 16 it is determined essentially by conditions in the "voltage regulator" circuit and by R_2. A consequence of this difference is that if the CM input voltage v_c were not zero, the collector current of Q_{1a} and Q_{1b} would change significantly in the circuit of Fig. 8, but would be essentially unaffected in that of Fig. 16. This effect is closely related to the improvement in CM rejection factor realized in the circuit of Fig. 16 and will be treated in detail later. Second, the collector currents of Q_{2a} and Q_{2b} and the CM output voltage v_{2c} may be approximately determined as follows. The current through R_6 is $(v_x + E_3)/(R_6 + R_7) = 2.06$ ma, if the base current of Q_3 is neglected. The collector current of Q_{2a} or Q_{2b} is therefore α_2 times half this current, in the absence of any unbalances, or very nearly 1 ma. The CM output voltage is therefore equal to the supply voltage E_1 less the drop in R_8, or 15 v.

With the basic first-order operation of the circuit of Fig. 16 understood, we can now proceed to a more detailed analysis to include the effects of unbalances.

5.2 Determination of the Common-Mode Loop Gain

Before embarking on the analysis of the complete amplifier circuit of Fig. 16, it is necessary to perform a subsidiary derivation to determine whether the CM loop gain is sufficiently high; first, for significant performance improvement to be realized; and second, to ensure the validity of the previous qualitative discussion of the circuit operation.

An expression for the CM loop gain G_c may easily be derived by consideration of a restricted form of the CM equivalent half-circuit. This is shown in Fig. 17, in which only the elements relevant to the CM feedback loop are included and all independent generators are omitted. If the loop is opened at some convenient point, say at the collector of Q_3, the CM loop gain may be obtained as the ratio i_y/i_x, where i_y is the current that results from an arbitrary injected current i_x. Straightforward analysis of the circuit of Fig. 17, in which z_{sc} is the CM source impedance, leads to

$$G_c \equiv \frac{i_y}{i_x} = \alpha_1\alpha_3 R_2 \Bigg/ 2\left(\frac{R_6 + R_7}{R_7}\right)\left(R_5 + \frac{R_6 \| R_7}{1 + \beta_3}\right) \qquad (5.2)$$

if $(1+\beta_1)(R_1+2R_3) \gg z_{ce}$, $2R_3 \gg R_1$, and $(1+\beta_2)2R_6 \gg R_2$. For the numerical values given in Fig. 16 these approximations are valid, and Eq. 5.2 then gives $G_c = 14.3$, or 23 db. It is seen that for high CM loop gain it is desirable to minimize R_5. The smallest realizable value of R_5 corresponds to the internal emitter resistance of Q_3; hence the choice $R_5 = 0.05$ k in Fig. 16 is reasonable.

The value $G_c = 14.3$ is high enough to vindicate the foregoing qualitative discussion, and more than an order of magnitude improvement in the appropriate performance parameters is to be expected.

5.3 Amplifier Analysis by the Sequential Method

Since the circuit of Fig. 16 is considerably more complex than that of Fig. 8, it is even more important to establish an orderly procedure directed toward the desired analysis goals and to introduce simplifications whenever possible. In accordance with the approach suggested in Section 3.2, it saves effort to relate the initial analysis to a composite equivalent half-circuit, which contains both the CM and DM half-circuits as special cases, as shown in Fig. 18. As in Fig. 11, e_1, j_1, etc., represent CM or DM independent generators and interaction generators, for which specific expressions will be substituted when the circuit equations are restricted to CM or to DM signals.

The ultimate analysis goals are to obtain the CM and DM output voltages, and the CM and DM input currents, as functions of the CM and DM signal voltages, the power supply voltages, and the internal independent generators, in the presence of unbalances in the homologous elements R_1, R_2, β_1, and β_2. From these results the performance parameters can be obtained. It will be assumed that there is no unbalance in R_8, since this would merely introduce an added complication which is trivial in principle.

The first step in the solution is to obtain, from Fig. 18, expressions for the input current, output voltage, and for the current in each element whose unbalance is to be considered. A corresponding interaction generator must be introduced for each such element. Thus expressions for i, i_1, i_2, i_3, and v_2 must be obtained as functions of v, E_1, E_2, E_3, e_1, e_2, e_3, e_4, j_1, j_2, and j_3. This is no small chore, and it is easy to become enmeshed in a web of seemingly interminable equations unless care is taken to keep the algebra under control. One way to break down the analysis into manageable pieces is to open the feedback loop and then determine two separate expressions for each desired quantity, in the following way. If the loop is broken at the collector of Q_3, as indicated in Fig. 18, a current (or voltage) i_z which exists in any other part of the circuit when the loop is closed may be obtained as

$$i_z = \frac{1}{1+G_c}\, i_z[i_x = 0] + \frac{G_c}{1+G_c}\, i_z[i_y = 0] \tag{5.3}$$

where $i_z[i_x = 0]$, $i_z[i_y = 0]$ are the values of i_z when $i_x = 0$, $i_y = 0$, respectively. This method is particularly appropriate since the first term on the right-hand side of Eq. 5.3 becomes negligible when the loop gain is very large, a result which later is helpful in making approximations.

Even when we use this technique, the expressions for the desired quantities in the circuit of Fig. 18 are quite lengthy, and so the procedure for only one of these, namely i_2, will be given as an example. The two components of i_2 necessary to insert in Eq. 5.3 are found to be

$$\begin{aligned} i_2[i_x = 0] = \Bigg[&(E_1 + E_3 - e_4) \\ &- \frac{R_7}{R_7 + (1+\beta_3)R_5}(E_3 - E_2 + e_3 + 2R_5 j_3) \\ &+ 2(1+\beta_2)[R_6 + R_7 \| (1+\beta_3)R_5]\left(\frac{\alpha_1(v - e_1)}{R_1 + 2R_3} + \frac{j_1}{1+\beta_1} \right. \\ &\left. - \frac{j_2}{1+\beta_2}\right)\Bigg]\left[\frac{1}{R_2 + 2(1+\beta_2)[R_6 + R_7 \| (1+\beta_3)R_5]}\right] \end{aligned} \tag{5.4}$$

$$\begin{aligned} i_2[i_y = 0] = \Bigg[&(E_1 + E_3 - e_4) - \frac{R_6 + R_7}{R_7}\Bigg(E_3 - E_2 + e_3 \\ &- 2[R_5 + R_6 \| R_7]\frac{j_3}{\beta_3}\Bigg)\Bigg]\frac{1}{R_2} \end{aligned} \tag{5.5}$$

When Eqs. 5.4 and 5.5 are substituted into Eq. 5.3, the result is an expression for i_2 in the circuit of Fig. 18 with the CM feedback loop closed. Similar procedures lead to corresponding expressions for the other quantities of interest.

The second step in the solution is to modify the various expressions to apply to the CM and to the DM equivalent half-circuits. Since the results are simpler for the DM half-circuit, these will be considered first. The composite equivalent half-circuit of Fig. 18 is valid for DM signals if R_3, R_6, R_7, β_3, and j_3 are made zero, and if the supply voltages E_1, E_2, and E_3 are short-circuited. In addition, since the CM feedback is inoperative for DM signals, G_c is zero (already ensured by setting $\beta_3 = 0$). These conditions lead to considerable simplifications in the expressions based on Eq. 5.3: in particular, only the $i_z[i_x = 0]$ term is required since G_c is zero. The quantities required are i, i_1, i_2, and i_3 with added subscript *do*, these

being the DM signals, in the absence of circuit unbalances, that are required to evaluate the interaction generators in the CM equivalent half-circuit. The quantities are obtained by the substitutions $e_1 = V_{1d}$, $e_2 = 0$, $e_3 = 0$, $e_4 = V_{2d}$, $j_1 = I_{1d}$, $j_2 = I_{2d}$, into Eq. 5.3 along with the other restrictions already noted. The results are

$$i_{do} = \frac{v_d - v_{1d}}{(1 + \beta_1)R_1} - \frac{I_{1d}}{1 + \beta_1} \tag{5.6}$$

$$i_{1do} = \frac{v_d - V_{1d}}{R_1} \tag{5.7}$$

$$i_{2do} = -\frac{V_{2d}}{R_2} \tag{5.8}$$

$$i_{3do} = -\alpha_1 \frac{v_d - V_{1d}}{R_1} - \frac{V_{2d}}{R_2} - \frac{I_{1d}}{1 + \beta_1} \tag{5.9}$$

It may be verified that Eq. 5.8 for i_{2do} is indeed derived from Eqs. 5.3 and 5.4.

In addition, the quantities i and v_2 are required with added subscript d, these being the DM signals, in the presence of circuit unbalances, that are required to evaluate the complete circuit performance parameters. The quantities are obtained by the substitutions $e_1 = V_{1d} + \delta R_1 i_{1co}$, $e_2 = \delta R_2 i_{2co}$, $e_3 = 0$, $e_4 = V_{2d}$, $j_1 = I_{1d} + \delta\beta_1 i_{co}$, $j_2 = I_{2d} + \delta\beta_2 i_{3co}$, into Eq. 5.3 along with the other restrictions already noted. The results are

$$i_d = i_{do} - \alpha_1 \frac{\delta\beta_1}{\beta_1} i_{co} - \frac{1}{1 + \beta_1} \frac{\delta R_1}{R_1} i_{1co} \tag{5.10}$$

$$\begin{aligned} v_{2d} = \beta_2 R_8 \left[\alpha_1 \frac{v_d - V_{1d}}{R_1} + \frac{I_{1d}}{1 + \beta_1} + \frac{V_{2d}}{R_2} - \frac{I_{2d}}{\beta_2} \right] \\ + \beta_2 R_8 \left[\alpha_1 \frac{\delta\beta_1}{\beta_1} i_{co} - \alpha_1 \frac{\delta R_1}{R_1} i_{1co} + \frac{\delta R_2}{R_2} i_{2co} - \frac{\delta\beta_2}{\beta_2} i_{3co} \right] \end{aligned} \tag{5.11}$$

Expressions for the various CM signals are obtained in a similar manner, but are more complicated since no elements can be omitted when the composite equivalent half-circuit of Fig. 18 is restricted to CM signals only. It is possible, however, to make some approximations when typical numerical values are considered. The two components of i_2, given by Eqs. 5.4 and 5.5, will again be chosen for an example of the procedure. It will be noted that some of the independent generator quantities, E_1, E_2, E_3, e_3, e_4, j_3, appear in both Eqs. 5.4 and 5.5, although others, e_1, j_1, j_2, appear only

in Eq. 5.4. When these two equations are substituted into Eq. 5.3 to obtain the actual value of i_2, it is found that the terms in the quantities that appear in both equations are dominated by the component from Eq. 5.5, since they are multiplied by G_c whereas those from Eq. 5.4 are not. Hence only terms in quantities which do not appear in Eq. 5.5 need be retained in Eq. 5.4. A similar argument applies when Eq. 5.3 is used to find the other CM currents.

The quantities required are i, i_1, i_2, and i_3 with added subscript co, these being the CM signals, in the absence of circuit unbalances, that are required to evaluate the interaction generators in the DM equivalent half-circuit. The quantities are obtained by the substitutions $e_1 = V_{1c}$, $e_2 = 0$, $e_3 = V_{3c}$, $e_4 = V_{2c}$, $j_1 = I_{1c}$, $j_2 = I_{2c}$, $j_3 = I_{3c}$ into Eq. 5.3 along with the approximations already noted. The results are

$$\begin{aligned} (1 + G_c)i_{co} = \left[\frac{v_c - V_{1c}}{(1 + \beta_1)(R_1 + 2R_3)} \right] \\ + G_c \left[\frac{(E_1 + E_3 - V_{2c}) - n(E_3 - E_2 + V_{3c}) + 2R_6 I_{3c}/\beta_3}{\beta_1 R_2} \right. \\ \left. - \frac{I_{1c}}{1 + \beta_1} + \frac{I_{2c}}{\beta_1(1 + \beta_2)} \right] \end{aligned} \tag{5.12}$$

$$\begin{aligned} (1 + G_c)i_{1co} = \left[\frac{v_c - V_{1c}}{R_1 + 2R_3} \right] \\ + G_c \left[\frac{(E_1 + E_3 - V_{2c}) - n(E_3 - E_2 + V_{3c}) + 2R_6 I_{3c}/\beta_3}{\alpha_1 R_2} \right. \\ \left. - \frac{I_{1c}}{\beta_1} + \frac{I_{2c}}{\alpha_1(1 + \beta_2)} \right] \end{aligned} \tag{5.13}$$

$$\begin{aligned} (1 + G_c)i_{2co} = \left[\alpha_1 \frac{v_c - V_{1c}}{R_1 + 2R_3} + \frac{I_{1c}}{1 + \beta_1} - \frac{I_{2c}}{1 + \beta_2} \right] \\ + G_c \left[\frac{(E_1 + E_3 - V_{2c}) - n(E_3 - E_2 + V_{3c}) + 2n(R_5 + R_6 \| R_7) I_{3c}/\beta_3}{R_2} \right] \end{aligned} \tag{5.14}$$

$$\begin{aligned} (1 + G_c)i_{3co} = \left[- \frac{R_2}{2(1 + \beta_2)R_6'} \left(\alpha_1 \frac{v_c - V_{1c}}{R_1 + 2R_3} \right. \right. \\ \left. \left. + \frac{I_{1c}}{1 + \beta_1} - \frac{E_1 + E_3 - V_{2c}}{R_2} \right) \right] \\ + G_c \left[\frac{E_3 - E_2 + V_{3c} - 2R_5 I_{3c}/\beta_3}{2(1 + \beta_2)R_7} - \frac{I_{2c}}{1 + \beta_2} \right] \end{aligned} \tag{5.15}$$

where
$$n \equiv \frac{R_6 + R_7}{R_7} \tag{5.16}$$

$$R_6' \equiv R_6 + R_7 \| (1 + \beta_3) R_5 \tag{5.17}$$

and in which certain other good approximations have been made, namely, $(1 + \beta_2)2R_6 \gg R_2$ and $R_7 \gg R_5$. It may be observed that n is the reciprocal of the potential divider ratio connected with the voltage regulator action and that R_6' is the effective resistance to ground seen by the second-stage common emitters.

In addition, the quantities i and v_2 are required with added subscript c, these being the CM signals, in the presence of circuit unbalances, that are required to evaluate the complete performance parameters. The quantities are obtained by the substitutions $e_1 = V_{1c} + \delta R_1 i_{1do}$, $e_2 = \delta R_2 i_{2do}$, $e_3 = V_{3c}$, $e_4 = V_{2c}$, $j_1 = I_{1c} + \delta\beta_1 i_{do}$, $j_2 = I_{2c} + \delta\beta_2 i_{3co}$, $j_3 = I_{3c}$, into Eq. 5.3 along with the approximations already noted. The results are

$$\begin{aligned}(1 + G_c)i_c = (1 + G_c)i_{co} &+ \left[-\frac{R_1}{(1 + \beta_1)(R_1 + 2R_3)} \frac{\delta R_1}{R_1} i_{1do} \right] \\ &+ G_c \left[-\frac{1}{1 + \beta_1} \frac{\delta\beta_1}{\beta_1} i_{do} - \frac{1}{\beta_1} \frac{\delta R_2}{R_2} i_{2do} + \frac{\alpha_2}{\beta_1} \frac{\delta\beta_2}{\beta_2} i_{3do} \right]\end{aligned} \tag{5.18}$$

$$\begin{aligned}(1 + G_c)v_{2c} = &\left[E_1 + \alpha_2 R_8 \frac{R_2}{2R_6'} \left(\alpha_1 \frac{v_c - V_{1c}}{R_1 + 2R_3} + \frac{I_{1c}}{1 + \beta_1} - \frac{E_1 + E_3 - V_{2c}}{R_2} \right) \right] \\ &+ G_c \left[E_1 - \alpha_2 R_8 \left(\frac{E_3 - E_2 + V_{3c}}{2R_7} + \frac{I_{2c}}{\beta_2} - \frac{I_{3c}}{\beta_3} \right) \right] \\ &+ \left[\alpha_2 R_8 \frac{R_2}{2R_6'} \left(\alpha_1 \frac{\delta\beta_1}{\beta_1} i_{do} - \alpha_1 \frac{R_1}{R_1 + 2R_3} \frac{\delta R_1}{R_1} i_{1do} + \frac{\delta R_2}{R_2} i_{2do} \right) \right] \\ &+ G_c \left[-\alpha_2 R_8 \frac{\delta\beta_2}{\beta_2} i_{3do} \right]\end{aligned} \tag{5.19}$$

The expressions for the various CM quantities may appear formidable, but actually the dominant terms can easily be related to the physical action of the circuit. For example, $(E_3 - E_2 + V_{3c})/R_7$ is the current in R_7, which is also the current in R_6, and so the current in R_8 is $\alpha_2(E_3 - E_2 + V_{3c})/2R_7$. The CM output voltage is therefore $[E_1 - \alpha_2 R_8(E_3 - E_2 + V_{3c})/2R_7] = 15$ v, as calculated in Section 5.1, and this is seen to be the dominant term in Eq. 5.19. The base current of Q_{2a} or Q_{2b} is $(E_3 - E_2 + V_{3c})/(1 + \beta_2)2R_7$, which is seen to be the dominant term in Eq. 5.15. The major contribution to i_{2co} in Eq. 5.14 is that of the three supply voltages and contains the factor $G_c/(1 + G_c)$; the contribution of the CM input voltage v_c contains the factor $1/(1 + G_c)$, however, and thus is negligible for large CM loop gain. Since i_{2co} is essentially equal to the collector current of Q_{1a} and Q_{1b} (except for the small i_{3co}), Eq. 5.14 explains why the first-stage operating current is determined primarily by the voltage regulator action and is little affected by the CM input voltage. Continued reasoning, as in Section 5.1, likewise identifies the dominant terms in Eqs. 5.12 and 5.13.

The third step in the solution is to eliminate the interaction generator terms between Eqs. 5.6 through 5.9 and Eqs. 5.18 and 5.19 and between Eqs. 5.12 through 5.15 and Eqs. 5.10 and 5.11. This procedure gives the CM and DM input currents and output voltages in terms of all the internal and external generators. It is not necessary to write out these complete expressions, since only some of the terms are required at one time for determination of the circuit performance parameters.

The fourth and final step in the solution is to determine the various circuit performance parameters from appropriate terms in the expressions for the CM and DM input currents and output voltages. The definitions of the various performance parameters listed in Table I are employed with slight modification and extension, in order to relate them to the two-stage amplifier under discussion. The gains refer to the output voltages v_{2c} and v_{2d} instead of to v_{1c} and v_{1d}; a third PS rejection factor $1/H_3 \equiv v_{2d}[E_3]/(-A_{dd})E_3$ is required; the CM internal independent generators include V_{2c}, I_{2c}, V_{3c}, and I_{3c} in addition to V_{1c} and I_{1c}; the DM internal independent generators include V_{2d} and I_{2d} in addition to V_{1d} and I_{1d}; and the minus signs are omitted because of the phase reversal in the second stage. Evaluation of the modified expressions, through use of the equations obtained in the third step of the solution, leads to the following results for the performance parameters of the two-stage amplifier of Fig. 16.

$$A_{cc} = \frac{\alpha_1 R_2}{(1 + G_c)(R_1 + 2R_3)} \frac{\alpha_2 R_8}{2R_6'} \tag{5.20}$$

$$\begin{aligned}A_{cd} = \frac{1}{1 + G_c} &\left[\frac{\alpha_1 R_2}{R_1} \frac{\alpha_2 R_8}{2R_6'} \left(\frac{1}{1 + \beta_1} \frac{\delta\beta_1}{\beta_1} - \frac{R_1}{R_1 + 2R_3} \frac{\delta R_1}{R_1} \right) \right] \\ &+ \frac{G_c}{1 + G_c} \left[\frac{\alpha_1 \alpha_2 R_8}{R_1} \left(\frac{\delta\beta_2}{\beta_2} \right) \right]\end{aligned} \tag{5.21}$$

$$A_{dd} = \frac{\alpha_1}{R_1}\beta_2 R_8 \tag{5.22}$$

$$\begin{aligned}A_{dc} = \frac{\alpha_1\beta_2 R_8}{(1+G_c)(R_1+2R_3)}\biggl(&\frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1} \\ &+ \frac{R_2}{(1+\beta_2)2R_6'}\frac{\delta\beta_2}{\beta_2}\biggr)\end{aligned} \tag{5.23}$$

$$\frac{1}{H_d} = \frac{R_1+2R_3}{R_1}\left[\left(\frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1} - \frac{R_1}{R_1+2R_3}\frac{\delta R_1}{R_1}\right) + G_c\left(\frac{2R_6'}{R_2}\frac{\delta\beta_2}{\beta_2}\right)\right] \tag{5.24}$$

$$\begin{aligned}\frac{1}{H_c} = \frac{R_1}{(1+G_c)(R_1+2R_3)}\biggl(&\frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1} \\ &+ \frac{R_2}{(1+\beta_2)2R_6'}\frac{\delta\beta_2}{\beta_2}\biggr)\end{aligned} \tag{5.25}$$

$$\begin{aligned}\frac{1}{H_1} = \frac{1}{1+G_c}&\left[\frac{R_1}{\alpha_1 R_2}\left(-\frac{R_2}{(1+\beta_2)2R_6'}\frac{\delta\beta_2}{\beta_2}\right)\right] \\ &+ \frac{G_c}{1+G_c}\left[\frac{R_1}{\alpha_1 R_2}\left(\frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right)\right]\end{aligned} \tag{5.26}$$

$$\begin{aligned}\frac{1}{H_2} = \frac{G_c}{1+G_c}\biggl[n\frac{R_1}{\alpha_1 R_2}\biggl(&\frac{R_2}{(1+\beta_2)2(R_6+R_7)}\frac{\delta\beta_2}{\beta_2} + \frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} \\ &+ \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\biggr)\biggr]\end{aligned} \tag{5.27}$$

$$\begin{aligned}\frac{1}{H_3} = \frac{1}{1+G_c}&\left[\frac{R_1}{\alpha_1 R_2}\left(-\frac{R_2}{(1+\beta_2)2R_6'}\frac{\delta\beta_2}{\beta_2}\right)\right] \\ &+ \frac{G_c}{1+G_c}\biggl[(1-n)\biggl(\frac{R_1}{\alpha_1 R_2}\frac{R_2}{(1+\beta_2)2R_6}\frac{\delta\beta_2}{\beta_2} + \frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} \\ &+ \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\biggr)\biggr]\end{aligned} \tag{5.28}$$

$$\begin{aligned}V_{di} = &-V_{1d} + \frac{R_1}{\alpha_1 R_2}V_{2d} + \frac{R_1}{\beta_1}I_{1d} - \frac{R_1}{\alpha_1\beta_2}I_{2d} \\ &- V_{1c}\left[\frac{1}{1+G_c}\left(\frac{R_2}{(1+\beta_2)2R_6'}\frac{\delta\beta_2}{\beta_2} + \frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{R_1}{R_1+2R_3} \\ &+ V_{2c}\left[\frac{1}{1+G_c}\left(\frac{R_2}{(1+\beta_2)2R_6'}\frac{\delta\beta_2}{\beta_2}\right) - \frac{G_c}{1+G_c}\left(\frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{R_1}{\alpha_1 R_2} \\ &- V_{3c}\left[\frac{G_c}{1+G_c}\left(\frac{R_2}{(1+\beta_2)2(R_6+R_7)}\frac{\delta\beta_2}{\beta_2} + \frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{nR_1}{\alpha_1 R_2} \\ &+ I_{1c}\left[\frac{1}{1+G_c}\left(\frac{R_2}{(1+\beta_2)2R_6'}\frac{\delta\beta_2}{\beta_2} + \frac{\delta R_2}{R_2}\right) - \frac{G_c}{1+G_c}\left(\frac{\delta\beta_1}{\beta_1} - \frac{\delta R_1}{R_1}\right)\right]\frac{R_1}{\beta_1} \\ &- I_{2c}\left[\frac{1}{1+G_c}\left(\frac{\delta R_2}{R_2}\right) - \frac{G_c}{1+G_c}\left(\frac{\delta\beta_2}{\beta_2} - \frac{\delta R_1}{R_1} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{R_1}{\alpha_1(1+\beta_2)} \\ &+ I_{3c}\left[\frac{G_c}{1+G_c}\left(\frac{R_5}{R_7}\frac{R_2}{2R_6}\frac{1}{1+\beta_2}\frac{\delta\beta_2}{\beta_2} + \frac{\delta R_2}{R_2} - \frac{\delta R_1}{R_1} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{2R_6}{R_2}\frac{R_1}{\alpha_1\beta_3}\end{aligned} \tag{5.29}$$

$$y_{cc} = \frac{1}{(1+G_c)(1+\beta_1)(R_1+2R_3)} \tag{5.30}$$

$$y_{dd} = \frac{1}{(1+\beta_1)R_1} \tag{5.31}$$

$$\begin{aligned}y_{cd} = &-\frac{1}{(1+G_c)(1+\beta_1)(R_1+2R_3)}\frac{\delta R_1}{R_1} \\ &- \frac{G_c}{(1+G_c)(1+\beta_1)R_1}\left(\alpha_2\frac{\delta\beta_2}{\beta_2} + \frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right)\end{aligned} \tag{5.32}$$

$$y_{dc} = -\frac{1}{(1+G_c)(1+\beta_1)(R_1+2R_3)}\left(\frac{\delta R_1}{R_1} + \alpha_1\frac{\delta\beta_1}{\beta_1}\right) \tag{5.33}$$

$$I_{de} = -\frac{G_c}{1+G_c}\left(\frac{\delta R_1}{R_1} + \alpha_1\frac{\delta\beta_1}{\beta_1}\right)\frac{(E_1+E_3) - n(E_3-E_2)}{\beta_1 R_2} \tag{5.34}$$

$$
\begin{aligned}
I_{ci} = &-\frac{1}{(1+G_c)(1+\beta_1)(R_1+2R_3)}V_{1c} \\
&\qquad\qquad - \frac{G_c}{(1+G_c)\beta_1 R_2}(V_{2c}+nV_{3c}) \\
&-\frac{G_c}{(1+G_c)}\left(I_{1c}-\frac{1}{1+\beta_2}I_{2c}-\frac{2R_6}{\beta_3R_2}I_{3c}\right)\frac{1}{\beta_1} \\
&+V_{1d}\left[\frac{1}{1+G_c}\left(\frac{R_1}{R_1+2R_3}\frac{\delta R_1}{R_1}\right)\right. \\
&\qquad\left.+\frac{G_c}{1+G_c}\left(\alpha_2\frac{\delta\beta_2}{\beta_2}+\frac{1}{1+\beta_1}\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{1}{(1+\beta_1)R_1} \\
&-V_{2d}\left[\frac{G_c}{1+G_c}\left(\alpha_2\frac{\delta\beta_2}{\beta_2}-\frac{\delta R_2}{R_2}\right)\right]\frac{1}{\beta_1R_2} \\
&+I_{1d}\left[\frac{G_c}{1+G_c}\left(\alpha_1\frac{\delta\beta_1}{\beta_1}-\alpha_2\frac{\delta\beta_2}{\beta_2}\right)\right]\frac{1}{\beta_1(1+\beta_1)}
\end{aligned}
\tag{5.35}
$$

$$
\begin{aligned}
I_{di} = &-\frac{1}{(1+\beta_1)R_1}V_{1d}-\frac{1}{1+\beta_1}I_{1d} \\
&+V_{1c}\left[\frac{1}{1+G_c}\left(\frac{\delta R_1}{R_1}+\alpha_1\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{1}{(1+\beta_1)(R_1+2R_3)} \\
&+(V_{2c}+nV_{3c})\left[\frac{G_c}{1+G_c}\left(\frac{\delta R_1}{R_1}+\alpha_1\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{1}{\beta_1R_2} \\
&+I_{1c}\left[\frac{G_c}{1+G_c}\left(\frac{1}{\beta_1}\frac{\delta R_1}{R_1}+\alpha_1\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{1}{1+\beta_1} \\
&-I_{2c}\left[\frac{G_c}{1+G_c}\left(\frac{\delta R_1}{R_1}+\alpha_1\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{1}{\beta_1(1+\beta_2)} \\
&-I_{3c}\left[\frac{G_c}{1+G_c}\left(\frac{\delta R_1}{R_1}+\alpha_1\frac{\delta\beta_1}{\beta_1}\right)\right]\frac{2R_6}{\beta_1\beta_3R_2}
\end{aligned}
\tag{5.36}
$$

Equations 5.20 through 5.36 describe the performance of the circuit of Fig. 16 when driven from a zero impedance signal source. Expressions for A_1, A_2, and V_{ci} have not been given, since these quantities relate to the CM equivalent input voltage, which is of little importance because of the very low CM gain.

Before interpretation of the general results, it is again useful to insert typical numerical values in order to determine the relative importance of the various terms. For the resistances and transistors in Fig. 16 that also appear in Fig. 8, the same values and the same unbalances are chosen, and they are listed in Table II. In order to maintain comparable operating conditions in the two circuits the supply voltages are different and the additional values displayed in Table IV will be used, together with the value $G_c = 14.3$ determined in Section 5.2. The directions of the unbalances in the second stage are taken opposite from those in the first-stage transistors, in order to illustrate the worst-case combinations.

Substitution of the numerical values into Eqs. 5.20 through 5.36 leads to the following results.

$$A_{cc} = 6.66 \times 10^{-3} \tag{5.37}$$

$$
\begin{aligned}
A_{cd} &= \frac{1}{1+G_c}[20.6(0.002+0.00005)] + \frac{G_c}{1+G_c}[-4.8] \\
&= -4.48
\end{aligned}
\tag{5.38}
$$

$$A_{dd} = 2.45 \times 10^3 \tag{5.39}$$

$$
\begin{aligned}
A_{dc} &= \frac{12.2}{1+G_c}[0.01+0.01+0.002-0.00086] \\
&= 1.6 \times 10^{-2}
\end{aligned}
\tag{5.40}
$$

$$\frac{1}{H_d} = 201[(0.002+0.00005)+G_c(-0.233)] \tag{5.41a}$$

$$= -\frac{1}{1.49 \times 10^{-3}} \tag{5.41}$$

$$\frac{1}{H_c} = \frac{1}{201(1+G_c)}[0.01+0.01+0.002-0.00086] \tag{5.42a}$$

$$= \frac{1}{1.46 \times 10^5} \tag{5.42}$$

$$
\begin{aligned}
\frac{1}{H_1} &= \frac{1}{1+G_c}[8.76\times 10^{-6}] + \frac{G_c}{1+G_c}[2.24\times 10^{-4}] \\
&= \frac{1}{4.75 \times 10^3}
\end{aligned}
\tag{5.43}
$$

$$
\begin{aligned}
\frac{1}{H_2} &= \frac{G_c}{1+G_c}[6.53\times 10^{-4}] \\
&= \frac{1}{1.64 \times 10^3}
\end{aligned}
\tag{5.44}
$$

$$\frac{1}{H_3} = \frac{1}{1+G_c}[8.76 \times 10^{-6}] + \frac{G_c}{1+G_c}[-4.28 \times 10^{-4}]$$

$$= -\frac{1}{2.5 \times 10^3} \tag{5.45}$$

$$\begin{aligned} V_{di} = &-V_{1d} + 0.0102V_{2d} + 0.002I_{1d} - 0.002I_{2d} \\ &- V_{1c}\left[\frac{1}{1+G_c}(0.0211)\right]0.005 \\ &+ V_{2c}\left[\frac{1}{1+G_c}(-0.00086) - \frac{G_c}{1+G_c}0.022\right]0.0102 \\ &- V_{3c}\left[\frac{G_c}{1+G_c}(0.0213)\right]0.0306 \\ &+ I_{1c}\left[\frac{1}{1+G_c}(0.00914) - \frac{G_c}{1+G_c}(0.11)\right]0.002 \\ &- I_{2c}\left[\frac{1}{1+G_c}(0.01) - \frac{G_c}{1+G_c}(-0.088)\right]0.002 \\ &+ I_{3c}\left[\frac{G_c}{1+G_c}(0.022)\right]0.004 \end{aligned} \tag{5.46a}$$

$$\begin{aligned} &= -10^{-3}(1000V_{1d} - 10.2V_{2d} - 2I_{1d} + 2I_{2d}) \\ &\quad - 10^{-3}(0.0069V_{1c} + 0.21V_{2c} + 0.61V_{3c}) \\ &\quad - 10^{-3}(0.206I_{1c} + 0.164I_{2c} - 0.082I_{3c}) \end{aligned} \tag{5.46b}$$

$$\begin{aligned} &= -10^{-3}(30 + 0.306 - 0.05 - 0.05) \\ &\quad - 10^{-3}(0.00207 + 0.063 + 0.183) \\ &\quad - 10^{-3}(0.0515 + 0.041 - 0.0205) \end{aligned} \tag{5.46c}$$

$$= -10^{-3}(30 + 0.53)$$

$$= -30.5 \text{ mv} \tag{5.46}$$

$$\begin{aligned} \frac{\partial V_{di}}{\partial T} &= 10^{-6}(200 + 0.204 + 5 + 5) \\ &\quad + 10^{-6}(0.0138 + 0.42 + 1.22) \\ &\quad + 10^{-6}(-5.15 - 4.1 + 2.05) \end{aligned} \tag{5.47a}$$

$$= 10^{-6}(200 + 4.65)$$

$$= 0.205 \text{ mv/°C} \tag{5.47}$$

$$y_{cc} = \frac{1}{15{,}700} \tag{5.48}$$

$$y_{dd} = \frac{1}{5.1} \tag{5.49}$$

$$y_{cd} = -\frac{1}{(1+G_c)1025}[-0.01] - \frac{G_c}{(1+G_c)5.1}[-0.098 + 0.002]$$

$$= \frac{1}{57} \tag{5.50}$$

$$y_{dc} = -\frac{1}{(1+G_c)1025}[-0.01 + 0.098]$$

$$= -\frac{1}{178{,}500} \tag{5.51}$$

$$I_{dc} = -\frac{G_c}{1+G_c}[-0.01 + 0.098]0.02$$

$$= -1.65\ \mu\text{a} \tag{5.52}$$

$$\begin{aligned} I_{ci} = &-\frac{1}{(1+G_c)}0.000975V_{1c} - \frac{G_c}{1+G_c}0.002(V_{2c} + 3V_{3c}) \\ &- \frac{G_c}{1+G_c}(I_{1c} - 0.02I_{2c} - 0.04I_{3c})0.02 \\ &+ V_{1d}\left[\frac{1}{1+G_c}(-0.00005) + \frac{G_c}{1+G_c}(-0.096)\right]0.2 \\ &- V_{2d}\left[\frac{G_c}{1+G_c}(-0.099)\right]0.002 \\ &+ I_{1d}\left[\frac{G_c}{1+G_c}(0.186)\right]0.0004 \end{aligned} \tag{5.53a}$$

$$\begin{aligned} &= -10^{-3}(0.0636V_{1c} + 1.87V_{2c} + 5.61V_{3c}) \\ &\quad - 10^{-3}(18.7I_{1c} - 0.374I_{2c} - 0.748I_{3c}) \\ &\quad - 10^{-3}(18V_{1d} - 0.185V_{2d} - 0.0764I_{1d}) \end{aligned} \tag{5.53b}$$

$$\begin{aligned} &= -10^{-3}(0.01908 + 0.561 + 1.683) \\ &\quad - 10^{-3}(4.68 - 0.0935 - 0.187) \\ &\quad - 10^{-3}(0.54 + 0.00555 - 0.00191) \end{aligned} \tag{5.53c}$$

$$= -7.21\ \mu\text{a} \tag{5.53}$$

$$\begin{aligned} \frac{\partial I_{ci}}{\partial T} &= 10^{-6}(0.127 + 3.74 + 11.2) \\ &\quad + 10^{-6}(-468 + 9.35 + 18.7) \\ &\quad + 10^{-6}(3.6 + 0.037 + 0.191) \end{aligned} \tag{5.54a}$$

$$= -0.421\ \mu\text{a/°C} \tag{5.54}$$

$$
\begin{aligned}
I_{di} = {} & -10^{-3}(196V_{1d} + 19.6I_{1d}) \\
& + 10^{-3}(0.00561V_{1c} + 0.164V_{2c} + 0.492V_{3c}) \\
& + 10^{-3}(1.8I_{1c} - 0.0322I_{2c} - 0.066I_{3c}) && (5.55a) \\
= {} & -10^{-3}(5.88 + 0.49) \\
& - 10^{-3}(-0.00168 - 0.0492 - 0.148) \\
& - 10^{-3}(-0.45 + 0.00805 + 0.0165) && (5.55b) \\
= {} & -5.75\ \mu a && (5.55)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial I_{di}}{\partial T} = {} & 10^{-6}(39.2 - 49) \\
& + 10^{-6}(-0.0112 - 0.328 - 0.984) \\
& + 10^{-6}(45 - 0.805 - 1.65) && (5.56a) \\
= {} & 0.0314\ \mu a/^\circ C && (5.56)
\end{aligned}
$$

It is fruitful to compare the foregoing expressions and numerical results for the circuit of Fig. 16 (Eqs. 5.20 through 5.56) with those for the circuit of Fig. 8 (Eqs. 3.24 through 3.63). We see that the desirable results of CM negative feedback developed in Section 4.2 are achieved in the circuit of Fig. 16: The CM gain A_{cc}, the CM rejection factor H_c, and the CM and CM-to-DM input admittances y_{cc} and y_{dc} are each improved by the factor $(1 + G_c)$ over their values in the absence of CM negative feedback (Eqs. 5.20, 5.25, 5.30, and 5.33).

It must be remembered that the presence of CM feedback is not the only difference between the circuits of Figs. 8 and 16. The addition of the second differential stage in Fig. 16 will make the results different from those for the circuit of Fig. 8 even if CM feedback is not employed. Thus if G_c is zero in Eq. 5.20 for the CM gain A_{cc}, the resulting expression, which is the CM gain in the circuit of Fig. 16 in the absence of CM feedback, differs from that for the circuit of Fig. 8 (Eq. 3.24) and is in fact less by the factor $\alpha_2R_s/2R_6' = 0.21$. This factor may be identified as the CM gain of the second differential stage, and its presence explains why A_{cc} for Fig. 16 (Eq. 5.37) is less than A_{cc} for Fig. 8 (Eq. 3.42) by a factor greater than $(1 + G_c)$. The CM rejection factor H_c for the circuit of Fig. 16, in the absence of CM feedback (Eq. 5.25 with $G_c = 0$) also differs from that for Fig. 8 (Eq. 3.27) because of an additional term produced by possible unbalance in the second stage. For the values chosen, however, this extra term is insignificant (Eq. 5.42*a*), and hence the numerical value of H_c for Fig. 16 (Eq. 5.42) is greater than that for Fig. 8 (Eq. 3.45) by a factor almost equal to $(1 + G_c)$. The actual factor is slightly larger than $(1 + G_c)$ because of the (fortuitous) partial compensating effect of the direction of the β unbalance in the second stage.

As anticipated in Section 4.2, the effects of CM feedback on other performance parameters of the circuit of Fig. 16 can be quite diverse. The discrimination factor $F = A_{dd}/A_{cc}$ for Fig. 16 is $(2.45 \times 10^3)/(6.66 \times 10^{-3}) = 3.68 \times 10^5$, and is larger by a factor 2.2×10^3 than that for Fig. 8. The high value of this factor is a result not only of CM feedback, but also of the reduced CM gain and the increased DM gain afforded by the second differential stage. On the other hand, the output fractional unbalance $U = 1/FH_c = -0.183\%$, although improved, is reduced in the circuit of Fig. 16 by only a modest factor from the value 1.27% in the circuit of Fig. 8. This is because the DM rejection factor for Fig. 16 is actually considerably poorer than that for Fig. 8 (Eqs. 3.44 and 5.41), a result largely offsetting the improved value of F. The poorer DM rejection factor for Fig. 16 can be traced directly to the β unbalance in the second stage (Eqs. 5.24 and 5.41*a*), an effect not present in the circuit of Fig. 8. Moreover, CM negative feedback, however great, is powerless to reduce the output fractional unbalance below a finite minimum. From Eqs. 5.20 and 5.24, $U = 1/FH_d = A_{cc}/A_{dd}H_d$ and tends to $(\delta\beta_2/\beta_2)/(1 + \beta_2)$ as G_c tends to infinity. The reason such a limit exists is that the seat of the interaction effect caused by unbalanced second-stage β's lies outside the CM feedback loop. A similar conclusion was reached by Birt[12] with respect to unbalances in harmonic generation in the second stage of a push-pull amplifier.

The DM equivalent input voltage V_{di} in the circuit of Fig. 16 is seen to be dominated by the unbalance in the first-stage base-emitter voltages (Eq. 5.46*c*), as was the case in the circuit of Fig. 8 (Eq. 3.50). It may hence seem that the formidable collection of terms in Eq. 5.29 can be drastically pruned. As was mentioned in Section 3.4, however, if accurate matching or compensation techniques are employed, this dominant term can be greatly reduced, thus exposing lesser contributions. The most important of these is the contribution from the unbalance in the second-stage base-emitter voltages V_{2d} (Eq. 5.46*c*), and it may be noted that the factor R_1/α_1R_2 which multiplies V_{2d} in Eq. 5.29 is identified with the first-stage DM gain. The interaction terms in Eq. 5.29 are displayed so that each coefficient is divided into two parts, one of which is multiplied by $1/(1 + G_c)$, and the other by $G_c/(1 + G_c)$. From Eq. 5.46*a* it is clear that when both parts are present the one multiplied by $1/(1 + G_c)$ is always negligible. This is a result of the particular numerical values chosen for illustration, however, and even slightly different values of component unbalances could nullify the present dominant part and expose the minor contributor.

The temperature drift of the DM equivalent input voltage, $\partial V_{di}/\partial T$, is of even greater interest than its initial value. From Eq. 4.57*a*, this performance parameter is again dominated by $\partial V_{1d}/\partial T$. If this contributor is largely eliminated by matching or compensation, however, those produced by I_{1d}, I_{1c}, I_{2d}, I_{2c}, and I_{3c} all become significant. It should be par-

ticularly noted that both the first- and second-order contributions from the second stage caused by I_{2d} and I_{2c} are not reduced by the gain of the first stage, but are of equal importance with the first-stage contributions of I_{1d} and I_{1c}. Use of silicon transistors in the first stage is the usual means of reducing drift caused by saturation currents; we see, however, that silicon transistors must be used in *all* stages in the circuit of Fig. 16 if drift from this cause is to be significantly reduced.

Similar remarks apply to the DM input current I_{di} and its temperature drift $\partial I_{di}/\partial T$ as to V_{di} and $\partial V_{di}/\partial T$. Both I_{di} and $\partial I_{di}/\partial T$ are dominated by the contributions of V_{1d} (Eqs. 5.55*b* and 5.56*a*), as was the case for the circuit of Fig. 8, unless matching or compensation exposes lesser contributions. There is again, however, a fortuitous and almost-complete cancellation of the terms produced by I_{1d} and I_{1c} in the expression for $\partial I_{di}/\partial T$. The CM input current I_{ci} and its temperature drift $\partial I_{ci}/\partial T$ in the circuit of Fig. 16 are dominated by the contributions of I_{1c} (Eqs. 5.53*c* and 5.54*a*), although the lesser terms differ considerably in origin from those in the corresponding equations for the circuit of Fig. 8 (Eqs. 3.60 and 3.61). The lesser terms become more important if silicon transistors are used.

The PS rejection factors in the circuit of Fig. 16 differ considerably both in nature and in magnitude from those in the circuit of Fig. 8. In the circuit of Fig. 8, two supply voltages E_1 and E_2 are required; the PS_1 rejection factor H_1 is infinite (Eq. 3.30), and the PS_2 rejection factor H_2 is equal to the CM rejection factor H_c (Eq. 3.31). In the circuit of Fig. 16, three supply voltages E_1, E_2, and E_3 are required; each possesses a finite rejection factor, and in addition none of them is even as large as H_2 for the circuit of Fig. 8 (Eqs. 3.45, 5.43, 5.44, and 5.45), although the operating conditions are the same. The consequent necessity for the provision of more stable supply voltages for the circuit of Fig. 16 may be considered the price paid for improved performance in other respects. The reason for the greater sensitivity to the supply voltages may be traced to the regulator action described in Section 5.1. The collector currents of the first-stage transistors are directly related to all three supply voltages, and any unbalances in the elements through which these currents flow will therefore introduce DM equivalent input voltages. It is seen from Eqs. 5.26 through 5.28 and 5.43 through 5.45 that the dominant terms in H_1, H_2, and H_3 are those produced by unbalances in R_1 and R_2.

The sensitivity of the DM equivalent input voltage to the supply voltage E_1, a second-order effect, has been put to supposedly good use as a means for correcting first-order unbalances.[5,13] To illustrate, suppose that in the circuit of Fig. 16 all unbalances are zero except for V_{1d}, the unbalance in the first-stage base-emitter voltages. The CM and all the PS rejection factors are then infinite, since all homologous circuit elements are balanced, and the DM equivalent input voltage V_{di} is equal to $-V_{1d}$, all other terms being zero in Eq. 5.29. Since the DM gain A_{dd} is 2.45×10^3 (Eq. 5.39), the DM output voltage v_{2d} in the absence of an externally applied signal would be -73.5 v if V_{1d}-30 mv. However, this would saturate the output and it is therefore necessary in effect to buck out the equivalent input voltage of -30 mv to bring the DM output voltage to zero. We may do this by purposely unbalancing R_2 so that the PS rejection factors, which are then no longer infinite, introduce the requisite bucking voltage. The required condition is that the total DM equivalent input voltage V_d be zero:

$$V_d = V_{di} + \frac{1}{H_1}E_1 + \frac{1}{H_2}E_2 + \frac{1}{H_3}E_3 = 0 \qquad (5.57)$$

Substitution of $V_{di} = -V_{1d}$ and the $\delta R_2/R_2$ terms from Eqs. 5.26 through 5.28 leads to

$$-V_{1d} + \frac{G_c}{1+G_c}\frac{R_1}{\alpha_1 R_2}\frac{(E_1+E_3) - n(E_3-E_2)}{R_2}\,\delta R_2 = 0 \qquad (5.58)$$

This relation is easily interpreted physically: $[(E_2 + E_3) - n(E_3 - E_2)]/R_2$ is the CM collector current of each first-stage transistor (see Section 5.1); δR_2 times this is the interaction generator DM voltage introduced by the unbalance in R_2. Hence the DM equivalent input voltage is this interaction voltage divided by the first-stage DM gain $\alpha_1 R_2/R_1$. Solution of Eq. 5.58 gives the requisite unbalance in R_2 as $\delta R_2/R_2 = 0.314$. The desired result of bringing the DM output voltage to zero has thus been accomplished, but, unfortunately, the CM rejection factor H_c is no longer infinite. The relevant expression, obtained from Eq. 5.25, is

$$\frac{1}{H_c} = \frac{R_1}{(1+G_c)(R_1+2R_3)}\frac{\delta R_2}{R_2} \qquad (5.59)$$

The percentage unbalance in R_2 is rather too large for this equation to be accurate, but at least an approximate value of H_c can be obtained as $H_c \approx 10^4$, or 80 db. This method of adjusting the DM output voltage is therefore not recommended, both because of the degraded CM rejection factor, and because the adjustment in any case is inherently dependent on the stability of the supply voltages.

The analysis of this chapter has so far led to expressions for the performance of the amplifier of Fig. 16 in the absence of signal source impedance. The inclusion of finite source impedances, and their effects on the overall amplifier performance, are discussed next.

5.4 Performance Parameters for the Combined Two-Stage Amplifier and Source

The effects of finite signal source impedances may be included in the results for the two-stage amplifier circuit of Fig. 16 in the same manner as they were in Section 3.3 for the single-stage circuit of Fig. 8. The source is represented by four open-circuit impedances and two independent voltage generators, and the transfer properties from the CM and DM source voltages v_{0c} and v_{0d} to the amplifier CM and DM input voltages v_c and v_d are as given in Eqs. 3.85 through 3.98. For the circuit of Fig. 16, we need only substitute the numerical values for the amplifier input admittances and currents. The overall performance from source voltages to amplifier output voltages is then obtained by combining the results for the amplifier with those for the source, as was done in Section 3.4 for the circuit of Fig. 8.

Since the procedure for the two-stage amplifier is the same as that for the single-stage circuit, the intermediate numerical results for the source will not be presented, and instead the final results for the overall performance will be stated by substitution into Eqs. 3.112 through 3.121. These equations give the quantities that appear in the relations

$$v_{2c} = A_{cct}\left(v_{0c} + V_{ct} + \frac{1}{H_{dt}} v_{0d}\right) \tag{5.60}$$

$$v_{2d} = A_{ddt}\left(v_{0d} + V_{dt} + \frac{1}{H_{ct}} v_{0c}\right) \tag{5.61}$$

where

$$V_{ct} = V_{cit} + \frac{1}{A_{1t}} E_1 + \frac{1}{A_{2t}} E_2 \tag{5.62}$$

$$V_{dt} = V_{dit} + \frac{1}{H_{1t}} E_1 + \frac{1}{H_{2t}} E_2 \tag{5.63}$$

The amplifier output voltages are v_{2c} and v_{2d}, and both the CM and DM gains are positive for the two-stage amplifier of Fig. 16. Expressions for V_{cit}, A_{1t}, and A_{2t} are not given since, as previously mentioned, the CM gain is so low that the CM equivalent input drift is of little interest.

The same numerical values for the source impedances as in Section 3.3, given in Table III, will be assumed. Since the amplifier DM input admittance y_{dd} for the circuit of Fig. 16 is the same as that for Fig. 8, the source DM "gain" $A_{dds} = 0.912$ as before. Since the amplifier CM input admittance y_{cc} is even smaller than in the single-stage circuit, the source CM "gain" A_{ccs} is even closer to unity than before. The results are as follows.

$$A_{cct} = A_{ccs}A_{cc} \approx A_{cc} \tag{5.64}$$

$$= 6.66 \times 10^{-3} \tag{5.65}$$

$$A_{ddt} = A_{dds}A_{dd} \tag{5.66}$$

$$= 0.912 \times 2.45 \times 10^3$$

$$= 2.23 \times 10^3 \tag{5.67}$$

$$\frac{1}{H_{dt}} = \frac{1}{H_{ds}} + \frac{A_{dds}}{A_{ccs}H_d} \tag{5.68}$$

$$= -\frac{1}{6.3} - \frac{0.912}{1.49 \times 10^{-3}}$$

$$= -\frac{1}{1.63 \times 10^{-3}} \tag{5.69}$$

$$\frac{1}{H_{ct}} = \frac{1}{H_{cs}} + \frac{A_{ccs}}{A_{dds}H_c} \tag{5.70}$$

$$= \frac{1}{1.67 \times 10^5} + \frac{1}{0.912 \times 1.46 \times 10^5}$$

$$= \frac{1}{7.4 \times 10^4} \tag{5.71}$$

$$\frac{1}{H_{1t}} = \frac{1}{H_{1s}} + \frac{1}{A_{dds}H_1} \tag{5.72}$$

$$= \frac{1}{7.43 \times 10^3} + \frac{1}{0.912 \times 4.75 \times 10^3}$$

$$= \frac{1}{2.73 \times 10^3} \tag{5.73}$$

$$\frac{1}{H_{2t}} = \frac{1}{H_{2s}} + \frac{1}{A_{dds}H_2} \tag{5.74}$$

$$= \frac{1}{2.48 \times 10^3} + \frac{1}{0.912 \times 1.59 \times 10^3}$$

$$= \frac{1}{0.915 \times 10^3} \tag{5.75}$$

$$\frac{1}{H_{3t}} = \frac{1}{H_{3s}} + \frac{1}{A_{dds}H_3} \tag{5.76}$$

$$= -\frac{1}{3.72 \times 10^3} - \frac{1}{0.912 \times 2.38 \times 10^3}$$

$$= -\frac{1}{1.37 \times 10^3} \tag{5.77}$$

$$V_{dit} = V_{dis} + \frac{1}{A_{dds}} V_{di} \tag{5.78}$$

$$= \left(2.52 - \frac{30.5}{0.912}\right) \text{mv}$$

$$= -30.9 \text{ mv} \tag{5.79}$$

$$\frac{\partial V_{dit}}{\partial T} = \left(-0.0368 + \frac{0.205}{0.912}\right) \text{mv/°C} \tag{5.80}$$

$$= 0.188 \text{ mv/°C} \tag{5.81}$$

$$F_t = \frac{A_{ddt}}{A_{cct}} \tag{5.82}$$

$$= 3.35 \times 10^5 \tag{5.83}$$

$$U_t = \frac{1}{F_t H_{dt}} \tag{5.84}$$

$$= -0.00183, \quad \text{or } -0.18\,\% \tag{5.85}$$

Examination of these results shows, for the circuit of Fig. 16, that the presence of the particular values of source resistance chosen has little effect on the CM gain A_{cct}, the DM gain A_{ddt}, the DM rejection factor H_{dt}, or consequently on the discrimination factor F_t or the output fractional unbalance U_t. Likewise, the chosen source has little effect on the DM equivalent input voltage V_{dit}, although its contribution to $\partial V_{dit}/\partial T$ is significant. The same remarks also apply to the results for the circuit of Fig. 8, as discussed in Section 3.4. However, all three PS rejection factors in the circuit of Fig. 16 are significantly affected by the source, a result which is to be expected because of the dependence of the amplifier CM and DM input currents on the supply voltages. The CM negative feedback does not alleviate these effects, and indeed is the cause of them. On the other hand, the CM rejection factor H_{ct} is larger by more than an order of magnitude than any of the PS rejection factors, even though the contribution from the source is significant. This result verifies the prediction of Section 4.3 that

the degradations in H_{ct} produced by both the amplifier and the source are reduced substantially by CM negative feedback.

It has been shown in the preceding sections that a specific configuration of a two-stage differential amplifier exhibits performance with respect to discrimination factor and CM rejection that is greatly improved by the presence of CM negative feedback. These benefits are obtained at the cost of greater sensitivity to the various PS voltages, however. As a supplementary topic, we consider in the next section some effects on the equivalent input DM drift voltage of transistor current gain temperature dependence, a problem ignored in the previous analysis.

5.5 Effects Due to Temperature Dependence of the Transistor Current Gains

In establishing the numerical values for the single-stage d-c differential amplifier displayed in Table II we mentioned that the temperature dependence of the transistor common-emitter current gains may be significant. This dependence has been neglected throughout the foregoing analyses of the single-stage and two-stage d-c differential amplifiers. Inclusion of the dependence in the analysis leads to considerable further complexity; it is nevertheless possible by qualitative reasoning to establish approximate expressions for the magnitude of the effects on the equivalent input DM temperature drift.

Let us reconsider the two-stage amplifier of Fig. 16. We have seen in Section 5.1 that the voltage regulator action of the CM feedback loop tends to keep constant the collector currents of the first-stage transistors Q_{1a} and Q_{1b}. For the numerical values chosen, each collector current is about 1 ma. If the rate of increase of the common-emitter current gain with temperature is taken to be 0.25 %/°C (within the typical range 0.2 to 0.5 %/°C described in Section 3.2), the resulting change of base current at constant collector current is $-(20\ \mu\text{a} \times 0.0025)/°\text{C} = -50$ na/°C, since the base current is approximately 20 μa for a current gain of 50 (neglecting the saturation current). Note that the base current *decreases* with rising temperature.

The base current temperature dependence we have derived is effectively a component of $\partial I_{ci}/\partial T$ and therefore will give rise to a corresponding component of equivalent input DM drift through any unbalance in the source resistances. In addition, there may be an unbalance in the current gain temperature dependences of the two first-stage transistors Q_{1a} and Q_{1b}. If they differ respectively by minus and plus 10% from the mean value 0.25 %/°C, there will be a component of $\partial I_{di}/\partial T$ equal to $+5$ na/°C. There will in turn be a corresponding component of equivalent input DM drift

through the balanced source resistances. The relevant relation is Eq. 3.98 which, when differentiated with respect to temperature, is

$$\frac{\partial V_{dis}}{\partial T} = -z_{dd}\frac{\partial I_{di}}{\partial T} - \frac{z_{dc} - z_{cc}z_{dd}y_{dc}}{1 + z_{cc}y_{cc}}\frac{\partial I_{ci}}{\partial T} \tag{5.86}$$

For the numerical values given in Tables II, III, and IV, $z_{dd} = 0.5$ k, $z_{dc} = -0.05$ k, and the z_{dc} term is dominant in the coefficient of $\partial I_{ci}/\partial T$, hence

$$\frac{\partial V_{dis}}{\partial T} = (-0.5 \times +5) - (-0.05 \times -50) = -5 \text{ mv/°C} \tag{5.87}$$

This contribution to the total equivalent input DM temperature drift $\partial V_{dit}/\partial T$ is considerably in excess of the value 0.188 mv/°C calculated as due to other effects (Eq. 5.81), and therefore is dominant for the numerical values chosen. It may be noted, however, that had the unbalance in the source resistances been in the opposite direction ($z_{dc} = +0.05$ k instead of -0.05 k), the contribution of current gain temperature dependence would have canceled out.

Effects arising from the current gain variations in the second differential stage may be investigated in a similar manner. The voltage regulator action of the CM feedback loop tends to keep the emitter currents of Q_{2a} and Q_{2b} constant at about 1 ma, and if the same β-dependence on temperature of 0.25%/°C is assumed, each base current will change at -50 na/°C. These changes must be accommodated by Q_{1a} and Q_{1b}, since the currents in R_{2a} and R_{2b} remain constant, so that the CM component of the first-stage base current increases at 1 na/°C for a β_1 of 50. This component of $\partial I_{ci}/\partial T$ and the component of $\partial I_{di}/\partial T$ produced by unbalance in second-stage β-variations are seen to be small compared with those produced by first-stage β-variations. Additional contributions to equivalent input DM temperature drift arise from the current variations in the resistances R_{1a} and R_{1b} caused by the first and second stages; however, the magnitudes of these contributions are small compared with those already calculated.

5.6 Conclusions

The basic properties of single-stage and two-stage differential amplifiers have been defined and discussed in the preceding chapters. Although detailed treatment has been presented only for transistor circuits, the methods are equally applicable to vacuum-tube amplifiers.

The values employed in the numerical examples have been chosen to illustrate the many properties and effects to be expected in differential amplifiers, and it must be remembered that the magnitudes of many of these

effects, and particularly their relative magnitudes, are extremely sensitive to the magnitudes and also to the directions of the arbitrarily assumed unbalances. Considerably improved performance may be obtained from the same amplifier configurations merely by choosing different transistors and different operating points. Some possibilities are discussed in the following.

Use of silicon instead of germanium transistors immediately alleviates problems produced by the saturation currents, since these are three or four orders of magnitude smaller in silicon than in germanium units. As a result, equivalent input DM voltage drifts from this source can be discounted for reasonably low source resistances, in spite of the stronger temperature dependence of the saturation currents in silicon transistors. Silicon transistors in matched pairs, mounted in one can for close thermal coupling, provide unbalanced base-emitter voltage temperature dependences as low as 5 or 10 μv/°C, a vast improvement over the 200 μv/°C employed in the numerical examples. Since this source of equivalent input DM drift has been seen to be important, considerably better drift performance can obviously be obtained by using matched pairs of transistors, particularly in the first stage.

Drift produced by current gain variations with temperature have been seen to be dominant, however, and a more suitable choice of operating point is helpful in reducing this effect. The basic temperature dependence of the current gain must be accepted, but if the operating collector current is reduced the temperature variations of the base current will be correspondingly reduced. Modern silicon transistors maintain reasonable magnitudes of current gain at collector currents as low as 5 or 10 μa, with resulting base current temperature dependences of about 1 na/°C, instead of the value 50 na/°C employed in our numerical examples. In the two-stage amplifier of Fig. 16, the collector currents of Q_{1a} and Q_{1b} are determined primarily by the load resistances R_2, and hence low collector currents are achieved by using high values of R_2. Additional benefits result, from a circuit point of view, since the first-stage DM gain and the CM loop gain are both increased. In order to minimize equivalent input effects originating in the second stage, it is also desirable to lower the operating currents of Q_{2a} and Q_{2b}, which is accomplished by increasing R_6 and R_7.

It is not possible to discuss here the multitude of modified, improved, and extended differential-amplifier circuits that have been developed for use not only as d-c amplifiers, but also as a-c amplifiers, and for many other system functions. Each such configuration poses an analysis and design problem in itself, and our intention has been to introduce, develop, and illustrate a general method for the analysis of unbalanced symmetrical circuits, a method that permits the treatment of such circuits in general with reasonable algebraic simplicity.

Appendix I

Proof of the sequential analysis method for symmetrical circuits with small-percentage unbalances

A formal proof of the sequential method for analyzing unbalanced symmetrical circuits may be developed as follows. A proof of the bisection theorem for balanced symmetrical circuits is included as a special case. Figure 19 shows the essential features of a symmetrical circuit in which a branch containing a self impedance z, a transfer impedance z_t, and an independent generator V is representative of each side, and z_d and z_c are representative of impedances that are, respectively, bridged between and common to each side. Although the circuit of Fig. 19 is symmetrical in form, it is not symmetrical in magnitude, and the unbalances are represented by antisymmetric increments $\pm\delta z$, δz_t in the typical elements z and z_t. Further, the internal independent generators may not be equal and are distinguished as V_a and V_b on sides a and b respectively. Similarly, the external driving sources v may be different on the two sides and are distinguished as v_a and v_b.

Mesh equations may be set up in terms of arbitrary mesh currents... i_i, i_j, ..., etc., where homologous currents may not be equal and are distinguished by subscripts a and b. The dependent generator in Fig. 19 is arbitrarily taken to be a function of a branch current $(i_u - i_v)$ on the same side. The mesh currents through the representative bridging impedance z_d are common to both sides and are given a subscript d instead of a or b. There is no loss of generality in showing a common node between the external independent generator v and the representative branch containing the impedance z.

The notation provides that the mesh equations for the complete circuit can be divided into two groups which are symmetrical in form, one for each side. The group for side a, with only the relevant terms displayed, is

$$
\begin{aligned}
\cdots + (z_{ii}' + z + \delta z)i_{ai} - (z + \delta z)i_{aj} \cdots\cdots - z_{ip}i_{dp} - z_{iq}i_{dq} - z_{ir}i_{ar} \cdots \\
= -(z_t + \delta z_t)(i_{au} - i_{av}) - V_a \\
\cdots - (z + \delta z)i_{ai} + (z_{jj}' + z + \delta z)i_{aj} \cdots\cdots - z_{jp}i_{dp} - z_{jq}i_{dq} - z_{jr}i_{ar} \cdots \\
= +(z_t + \delta z_t)(i_{au} - i_{av}) + V_a - v_a \\
\cdots\cdots - z_{pi}i_{ai} \cdots\cdots - z_{pj}i_{aj} \cdot + (2z_{pp}' + z_d)i_{dp} - z_d i_{dq} \cdots \\
= \cdots - z_{pi}i_{bi} - z_{pj}i_{bj} \cdots \\
\cdots\cdots - z_{qi}i_{ai} \cdots\cdots - z_{qj}i_{aj} \cdot - z_d i_{dp} + (2z_{qq}' + z_d)i_{dq} \cdots \\
= \cdots - z_{qi}i_{bi} - z_{qj}i_{bj} \cdots \\
\cdots + (z_{rr}' + z_c)i_{ar} \cdots\cdots = -z_c i_{br}
\end{aligned}
\tag{A.1}
$$

In this group $z_{ii}' \equiv [z_{ii} - (z + \delta z)]$, where z_{ii} is the total self impedance of mesh i; similarly, $z_{jj}' \equiv [z_{jj} - (z + \delta z)]$, and $z_{rr}' \equiv (z_{rr} - z_c)$. Meshes p and q each lie half in side a and half in side b, hence z_{pp} is defined as the total self impedance in one-half of mesh p, and then $z_{pp}' \equiv (z_{pp} - z_d/2)$; similarly, $z_{qq}' \equiv (z_{qq} - z_d/2)$.

The corresponding group of mesh equations for side b may be written directly from that for side a merely by changing subscript a to subscript b, replacing δz, δz_t by $-\delta z$, $-\delta z_t$, and replacing i_{dp}, i_{dq} by $-i_{dp}$, $-i_{dq}$ (since the currents in meshes p and q are in the opposite relative sense with respect to sides a and b). If the first pair of equations is rearranged to contain only δz, δz_t, V, and v terms on the right-hand side, and if the corresponding equations for sides a and b are interleaved, the resulting set of equations for the complete circuit is

$$
\begin{aligned}
\cdots + (z_{ii}' + z)i_{ai} - z\,i_{aj} \cdots\cdots - z_{ip}i_{dp} - z_{iq}i_{dq} - z_{ir}i_{ar} \cdots \\
= -\delta z(i_{ai} - i_{aj}) - \delta z_t(i_{au} - i_{av}) - V_a \\
\cdots + (z_{ii}' + z)i_{bi} - z\,i_{bj} \cdots\cdots + z_{ip}i_{dp} + z_{iq}i_{dq} - z_{ir}i_{br} \cdots \\
= +\delta z(i_{bi} - i_{bj}) + \delta z_t(i_{bu} - i_{bv}) - V_b \\
\cdots - z\,i_{ai} + (z_{jj}' + z)i_{aj} \cdots\cdots - z_{jp}i_{dp} - z_{jq}i_{dq} - z_{jr}i_{ar} \cdots \\
= +\delta z(i_{ai} - i_{aj}) + \delta z_t(i_{au} - i_{av}) + V_a + v_a \\
\cdots - z\,i_{bi} + (z_{jj}' + z)i_{bj} \cdots\cdots + z_{jp}i_{dp} + z_{jq}i_{dq} - z_{jr}i_{br} \cdots \\
= -\delta z(i_{bi} - i_{bj}) - \delta z_t(i_{bu} - i_{bv}) + V_b + v_b \\
\cdots - z_{pi}i_{ai} - z_{pj}i_{aj} \cdots + (2z_{pp}' + z_d)i_{dp} - z_d i_{dq} \cdots \\
= \cdots - z_{pi}i_{bi} - z_{pj}i_{bj} \cdots \\
\cdots - z_{pi}i_{bi} - z_{pj}i_{bj} \cdots - (2z_{pp}' + z_d)i_{dp} + z_d i_{dq} \cdots \\
= \cdots - z_{pi}i_{ai} - z_{pj}i_{aj} \cdots \\
\cdots - z_{qi}i_{ai} - z_{qj}i_{aj} \cdots - z_d i_{dp} + (2z_{qq}' + z_d)i_{dq} \cdots \\
= \cdots - [z_{qi}i_{bi} - z_{qj}i_{bj} \cdots
\end{aligned}
$$

$$
\begin{aligned}
\cdots - z_{qi}i_{bi} - z_{qj}i_{bj} \cdots + z_d i_{dp} - (2z_{qq}' + z_d)i_{dq} \cdots \\
= \cdots - z_{qi}i_{ai} - z_{qj}i_{aj} \cdots \\
\cdots + (z_{rr}' + z_c)i_{ar} \cdots = -z_c i_{br} \\
\cdots + (z_{rr}' + z_c)i_{br} \cdots = -z_c i_{ar}
\end{aligned}
\tag{A.2}
$$

The next step is to convert this set of equations from representation in terms of sides a and b into representation in terms of the CM and DM currents and voltages. This is accomplished by addition and subtraction of each pair of equations in the set. At the same time, representative CM and DM currents i_c and i_d are defined by

$$i_c \equiv \frac{i_a + i_b}{2}, \qquad i_d \equiv \frac{i_a - i_b}{2} \tag{A.3}$$

and the representative unbalanced independent generators are replaced by CM and DM equivalents defined by

$$V_c \equiv \frac{V_a + V_b}{2}, \qquad V_d \equiv \frac{V_a - V_b}{2} \tag{A.4}$$

$$v_c \equiv \frac{v_a + v_b}{2} \qquad v_d \equiv \frac{v_a - v_b}{2} \tag{A.5}$$

This procedure results in a set of equations that can be divided into a CM group and a DM group:

$$
\begin{aligned}
\cdots + (z_{ii}' + z)i_{ci} - z\,i_{cj} \cdots - z_{ir}i_{cr} \cdots \\
= -\delta z(i_{di} - i_{dj}) - \delta z_t(i_{du} - i_{dv}) - V_c \\
\cdots - z\,i_{ci} + (z_{jj}' + z)i_{cj} \cdots - z_{jr}i_{cr} \cdots \\
= +\delta z(i_{di} - i_{dj}) + \delta z_t(i_{du} - i_{dv}) + V_c + v_c \\
\cdots + (z_{rr}' + 2z_c)i_{cr} \cdots = 0
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
\cdots + (z_{ii}' + z)i_{di} - z\,i_{dj} \cdots\cdots - z_{ip}i_{dp} - z_{iq}i_{dq} - z_{ir}i_{dr} \cdots \\
= -\delta z(i_{ci} - i_{cj}) - \delta z_t(i_{cu} - i_{cv}) - V_d \\
\cdots - z\,i_{di} + (z_{jj}' + z)i_{dj} \cdots\cdots - z_{jp}i_{dp} - z_{jq}i_{dq} - z_{jr}i_{dr} \cdots \\
= +\delta z(i_{ci} - i_{cj}) + \delta z_t(i_{cu} - i_{cv}) + V_d + v_d \\
\cdots - z_{pi}i_{di} - z_{pj}i_{dj} \cdots + \left(z_{pp}' + \frac{z_d}{2}\right)i_{dp} - \frac{z_d}{2}\,i_{dq} \cdots \\
= 0
\end{aligned}
$$

$$\cdots - z_{qi}i_{di} - z_{qj}i_{dj} \cdots - \frac{z_d}{2} i_{dp} + \left(z_{qq}' + \frac{z_d}{2}\right) i_{dq} \cdots\cdots$$

$$= 0$$

$$\cdots + z_{rr}' i_{dr} \cdots = 0 \qquad \text{(A.7)}$$

There are several conclusions to be drawn from these two groups of equations. First, suppose that the circuit elements, though not necessarily the independent generators, in sides a and b of the original circuit are symmetrical in magnitude as well as in form, so that δz and δz_t are each zero. Equations A.6 then permit solution for the CM currents i_c as functions of the CM generators V_c and v_c. However, the reduced Eqs. A.6 could have been written immediately from the original circuit in Fig. 19 by considering only one side, by writing a subscript c instead of a or b, replacing z_c by $2z_c$, and setting $z_d = \infty$. This last requirement results from the absence of the currents i_{dp} and i_{dq} in *any* of Eqs. A.6; hence these currents can be set equal to zero by making z_d infinite. Similarly, Eqs. A.7 permit solution for the CM currents i_d as functions of the DM generators V_d and v_d. However, the reduced Eqs. A.7 could have been written immediately from the original circuit by considering only one side, writing a subscript d instead of a or b, setting $z_c = 0$, and replacing z_d by $z_d/2$ and connecting the center tap of z_d to ground. This special case therefore constitutes a proof of the bisection theorem for symmetrical and balanced circuits, in which the analysis may be based on a "common-mode half-circuit" and a "differential-mode half-circuit."

Next, suppose that the original circuit is not balanced, in which case the unbalance terms on the right hand sides of Eqs. A.6 and A.7 must be retained. It is seen that element unbalances introduce "interaction" terms between the CM and DM signals. Physical interpretation of Eqs. A.6 and A.7 shows that the interaction terms in Eq. A.6 can be accounted for by fictitious generators introduced at appropriate places in the CM half-circuit. These fictitious generators are independent as far as the CM signals are concerned but are dependent on the DM signals. Analogously, the interaction terms in Eq. A.7 may be accounted for by fictitious generators introduced in the CM half-circuit. Thus the bisection theorem can still be invoked and the complete solution obtained for the unbalanced symmetrical circuit in terms of the various interaction generators. Figure 20 shows the CM and DM half-circuits including the fictitious generators that account for the unbalances.

However, the magnitudes of the interaction generators are not known explicitly: Eqs. A.6 cannot be solved unless $(i_{di} - i_{dj})$, etc., are known from the solution of Eqs. A.7, and Eqs. A.7 cannot be solved unless $(i_{ci} - i_{cj})$, etc., are known from the solution of Eqs. A.6. Thus simultaneous solution of Eqs. A.6 and A.7 is required. To simplify this process it is assumed that δz_t, V_c, and V_d are zero so that only the unbalance δz is to be considered. No loss of generality is incurred by this simplification, since the process can in principle be repeated with inclusion of these terms. To solve directly for the current differences $(i_{ci} - i_{cj})$, $(i_{di} - i_{dj})$, etc., it is convenient to rearrange Eqs. A.6 and A.7 with the definitions

$$i_{cm} \equiv i_{ci} - i_{cj} \qquad i_{cn} \equiv i_{ci} + i_{cj} \qquad \text{(A.8)}$$

$$i_{dm} \equiv i_{di} - i_{dj} \qquad i_{dn} \equiv i_{di} + i_{dj} \qquad \text{(A.9)}$$

The relevant terms in the resulting equations for meshes i and j are

$$\cdots + \left(\frac{z_{ii}'}{2} + z\right) i_{cm} + \left(\frac{z_{ii}'}{2}\right) i_{cn} \cdots - z_{ir}i_{cr} \cdots = -\delta z\, i_{dm}$$

$$\cdots - \left(\frac{z_{jj}'}{2} + z\right) i_{cm} + \left(\frac{z_{jj}'}{2}\right) i_{cn} \cdots - z_{jr}i_{cr} \cdots = +\delta z\, i_{dm} + v_c \qquad \text{(A.10)}$$

$$\cdots + \left(\frac{z_{ii}'}{2} + z\right) i_{dm} + \left(\frac{z_{ii}'}{2}\right) i_{dn} \cdots - z_{ip}i_{dp} \cdots = -\delta z\, i_{cm}$$

$$\cdots - \left(\frac{z_{jj}'}{2} + z\right) i_{dm} + \left(\frac{z_{jj}'}{2}\right) i_{dn} \cdots - z_{jp}i_{dp} \cdots = +\delta z\, i_{cm} + v_d \qquad \text{(A.11)}$$

A further simplification is to add the first equation to the second in each group, and the resulting two groups are then

$$\cdots \left(\frac{z_{ii}'}{2} + z\right) i_{cm} \quad + \quad \left(\frac{z_{ii}'}{2}\right) i_{cn} \cdots\cdots\cdots\cdots - z_{ir}i_{cr} \cdots = -\delta z\, i_{dm}$$

$$\cdots \left(\frac{z_{ii}' - z_{jj}'}{2}\right) i_{cm} + \left(\frac{z_{ii}' + z_{jj}'}{2}\right) i_{cn} \cdots - (z_{ir} + z_{jr}) i_{cr} \cdots = v_c$$

$$\cdots + (z_{rr}' + 2z_c) i_{cr} \cdots = 0 \qquad \text{(A.12)}$$

$$\cdots \left(\frac{z_{ii}'}{2} + z\right) i_{dm} \quad + \quad \left(\frac{z_{ii}'}{2}\right) i_{dn} \cdots\cdots\cdots\cdots - z_{ip}i_{dp} \cdots = -\delta z\, i_{cm}$$

$$\cdots \left(\frac{z_{ii}' - z_{jj}'}{2}\right) i_{dm} + \left(\frac{z_{ii}' + z_{jj}'}{2}\right) i_{dn} \cdots - (z_{ip} + z_{jp}) i_{dp} \cdots = v_d$$

$$\cdots + z_{rr}' i_{dr} \cdots = 0 \qquad \text{(A.13)}$$

The solution for i_{cm} from Eq. A.12 is then

$$\Delta_c i_{cm} = -\delta z\, i_{dm}\, \Delta_{cim} + v_c\, \Delta_{cjm} \tag{A.14}$$

where Δ_c, Δ_{cim}, and Δ_{cjm} are the CM half-circuit determinant and the appropriate minors. Similarly, the solution for i_{dm} from Eq. A.13 is

$$\Delta_d i_{dm} = -\delta z\, i_{cm}\, \Delta_{dim} + v_d\, \Delta_{djm} \tag{A.15}$$

If now i_{dmo} and i_{cmo} are respectively defined as the values of i_{dm} and i_{cm} in the *balanced* symmetrical circuit ($\delta z = 0$), then simultaneous solution of Eqs. A.14 and A.15 yields

$$\Delta_c i_{cm} = \frac{-\delta z\, i_{dmo}\, \Delta_{cim} + v_c\, \Delta_{cjm}}{1 - (\delta z)^2\, \Delta_{cim}\, \Delta_{dim}/\Delta_c\, \Delta_d} \tag{A.16}$$

$$\Delta_d i_{dm} = \frac{-\delta z\, i_{cmo}\, \Delta_{dim} + v_d\, \Delta_{djm}}{1 - (\delta z)^2\, \Delta_{cim}\, \Delta_{dim}/\Delta_c\, \Delta_d} \tag{A.17}$$

These results indicate that if δz is small enough, then i_{dm} in Eq. A.14 can be replaced by i_{dmo}, and i_{cm} in Eq. A.15 can be replaced by i_{cmo}. With this proviso, the currents in the interaction terms in Eqs. A.6 and A.7 and in Fig. 20 can be given an additional subscript *o*. This means that the circuit can be first solved as though it were balanced, and the resulting balanced branch currents then employed for the fictitious generators in the unbalanced circuit. Thus the right-hand sides of Eqs. A.6 and A.7 contain terms that are all known and the unbalanced circuit can be solved separately for the CM and DM signals.

It remains to determine how small the unbalance must be for this procedure to be valid. The necessary condition is that the denominator of Eq. A.16 or A.17 should be close to unity. How small δz has to be can be determined from consideration of the relevant determinants. From Eq. A.12 or A.13, the circuit determinant Δ may be written

$$\Delta = z\, \Delta_{im} + \Delta' \tag{A.18}$$

where the z-dependence of Δ is explicit and neither Δ_{im} nor Δ' contains z, and where the subscript c or d is applicable as appropriate. Formulation of the quantity $\delta z\, \Delta_{im}/\Delta$ from Eq. A.18 gives

$$\frac{\delta z\, \Delta_{im}}{\Delta} = \frac{\delta z}{z}\, \frac{\Delta - \Delta'}{\Delta} \tag{A.19}$$

It follows from Eq. A.19 that if $(\Delta - \Delta')/\Delta$ is less than unity, then the denominator of Eq. A.16 or Eq. A.17 is close to unity if $\delta z/z \ll 1$. To examine the magnitude of $(\Delta - \Delta')/\Delta$, consider Eq. A.18. In most

cases Δ' will be of the same sign as Δ, and smaller in magnitude, hence $0 < (\Delta - \Delta')/\Delta < 1$. If Δ' is of opposite sign to Δ, or of the same sign but larger in magnitude, $(\Delta - \Delta')/\Delta$ could be greater than unity, but then the value of z would be close to that necessary to make $\Delta = 0$ and the circuit unstable. In any case, if the value of z is such that the circuit is not close to oscillation, the condition $\delta z/z \ll 1$ is usually sufficient to make $(\delta z)^2\, \Delta_{cim}\, \Delta_{dim}/\Delta_c \Delta_d \ll 1$ and ensure the validity of the method.

It has thus been shown that an unbalanced symmetrical circuit may be analyzed sequentially by bisection into a CM half-circuit and a DM half-circuit, in which small-percentage unbalances in homologous elements may be accounted for by the introduction of fictitious generators that are functions of the signals present when the circuit is balanced. A precisely analogous analysis on the node instead of the mesh basis leads to corresponding results for unbalanced admittances or independent current generators.

Appendix II

The common-mode rejection factor of a single-stage vacuum-tube a-c differential amplifier

It was pointed out in Section 4.1 that the presence of finite output resistances in tubes and transistors usually prevents the CM rejection factor from becoming infinite, even when the coupling impedance is made infinite. To verify this conclusion, and to provide an illustration of the application of the sequential analysis method to tube circuits, an expression for the CM rejection factor of a single-stage vacuum-tube a-c differential amplifier will be derived.

The circuit to be considered is shown in Fig. 5. Let the two tubes be characterized by unbalanced amplification factors and plate resistances as follows:

$$\mu_a = \mu + \delta\mu$$

$$r_{pa} = r_p + \delta r_p$$

$$\mu_b = \mu - \delta\mu$$

$$r_{pb} = r_p - \delta r_p$$

It is assumed that the two load resistances are exactly balanced.

Since the a-c performance alone is to be considered, each tube may be represented by the usual small-signal equivalent circuit containing the plate resistance in series with the μ voltage generator. The composite equivalent half-circuit of the amplifier is as shown in Fig. 21, where the interaction generators e_1 and e_2 account for unbalances in the tube amplification factors and plate resistances.

The analysis problem is to find the DM output voltage v_{1d} in terms of the DM input voltage v_d and the CM input voltage v_c. From Fig. 21,

the current i_p is given by

$$i_p = \frac{\mu v - e_1 + e_2}{r_p + R_L + (1+\mu)2R} \tag{A.20}$$

and the output voltage v_1 is obtained as $v_1 = -R_L i_p$, or

$$v_1 = -\frac{R_L}{r_p + R_L + (1+\mu)2R}(\mu v - e_1 + e_2)$$

This result is made applicable to DM signals by adding the subscript d and setting $R = 0$:

$$v_{1d} = -\frac{R_L}{r_p + R_L}(\mu v_d - e_1 + e_2) \tag{A.21}$$

The interaction terms are given by

$$e_1 = \delta r_p i_{pco} \tag{A.22}$$

$$e_2 = \delta\mu\, v_{gco} \tag{A.23}$$

where i_{pco} and v_{gco} are respectively the CM values of the plate current and the grid-cathode voltage in the absence of unbalances. The expression for i_{pco} is obtained by making Eq. A.20 applicable to CM signals in the absence of unbalances, that is, by adding the subscript co and setting $e_1 = e_2 = 0$:

$$i_{pco} = \frac{\mu}{r_p + R_L + (1+\mu)2R} v_c \tag{A.24}$$

The expression for v_{gco} is obtained from Fig. 21 as

$$v_{gco} = v_c - 2R i_{pco} = \frac{r_p + R_L + 2R}{r_p + R_L + (1+\mu)2R} v_c \tag{A.25}$$

Elimination of i_{pco} and v_{gco} from Eqs. A.22 through A.25 and substitution for e_1 and e_2 into Eq. A.21 gives

$$v_{1d} = -\frac{R_L}{r_p + R_L}\left[\mu v_d - \frac{\mu\delta r_p - (r_p + R_L + 2R)\delta\mu}{r_p + R_L + (1+\mu)2R} v_c\right]$$

Rearrangement of this result gives

$$v_{1d} = -\frac{\mu R_L}{r_p + R_L}\left[v_d + \frac{r_p+R_L+2R}{r_p+R_L+(1+\mu)2R}\left(\frac{\delta\mu}{\mu} - \frac{r_p}{r_p+R_L+2R}\,\frac{\delta r_p}{r_p}\right)v_c\right] \tag{A.26}$$

The CM rejection factor is then the reciprocal of the coefficient of v_c in Eq. A.26, or

$$\frac{1}{H_c} = \frac{r_p + R_L + 2R}{r_p + R_L + (1+\mu)2R}\left(\frac{\delta\mu}{\mu} - \frac{r_p}{r_p + R_L + 2R}\frac{\delta r_p}{r_p}\right) \quad \text{(A.27)}$$

A special case of interest is the value of the CM rejection factor when the coupling resistance R tends to infinity:

$$\left.\frac{1}{H_c}\right|_{R\to\infty} = \frac{\delta\mu}{\mu(1+\mu)} \quad \text{(A.28)}$$

The conclusion of Parnum,[6] Andrew,[10] and Klein[11] is thus verified: Unbalanced tube amplification factors lead to a finite CM rejection factor even when the coupling impedance is infinite.

Also of special interest is the value of the CM rejection factor when the plate resistance r_p tends to infinity, which occurs when pentode tubes are used. Since the μ will also tend to infinity, it is necessary to recast Eq. A.27 in terms of the average transconductance g_m by use of

$$\mu = g_m r_p$$

from which, since the percentage unbalances are small,

$$\frac{\delta\mu}{\mu} = \frac{\delta g_m}{g_m} + \frac{\delta r_p}{r_p} \quad \text{(A.29)}$$

Substitution of Eq. A.29 into Eq. A.27 gives

$$\frac{1}{H_c} = \frac{1+k}{1+k+2g_mR}\left(\frac{\delta g_m}{g_m} + \frac{k}{1+k}\frac{\delta r_p}{r_p}\right) \quad \text{(A.30)}$$

where

$$k \equiv \frac{R_L + 2R}{r_p} \quad \text{(A.31)}$$

If now r_p tends to infinity, k tends to zero, and

$$\left.\frac{1}{H_c}\right|_{r_p\to\infty} = \frac{1}{1+2g_mR}\frac{\delta g_m}{g_m} \quad \text{(A.32)}$$

It may be noted that in this case the CM rejection factor tends to infinity as the coupling impedance tends to infinity, even in the presence of unbalanced tube transconductances.

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Figures

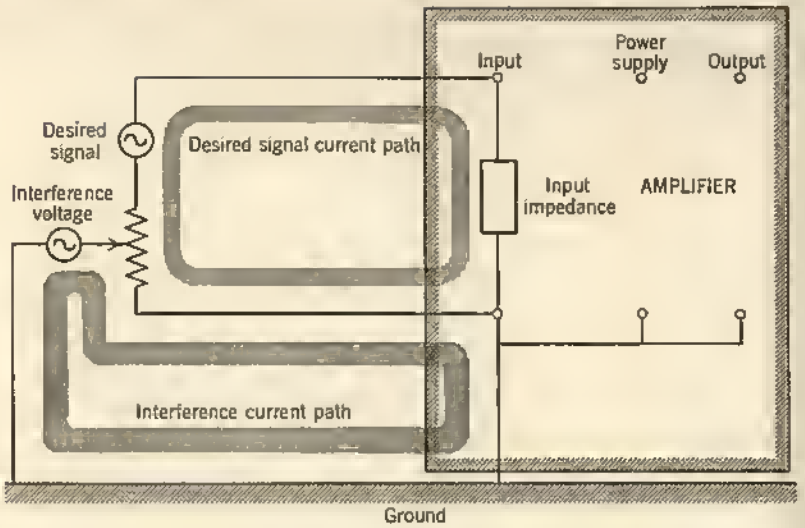


Fig. 1. Instrumentation system employing an amplifier with a single-ended input. In spite of a high amplifier input impedance, interference voltages can generate large spurious input signals.

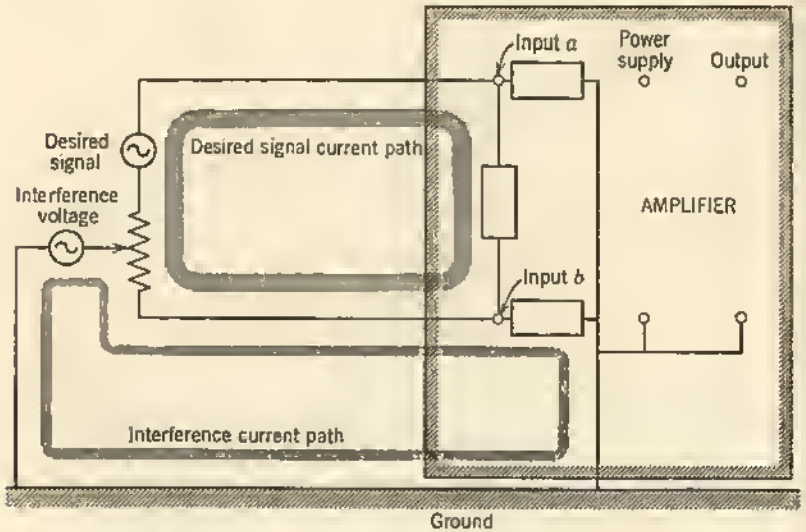


Fig. 2. Instrumentation system employing an amplifier with a floating input. Currents due to interference voltages are greatly reduced.

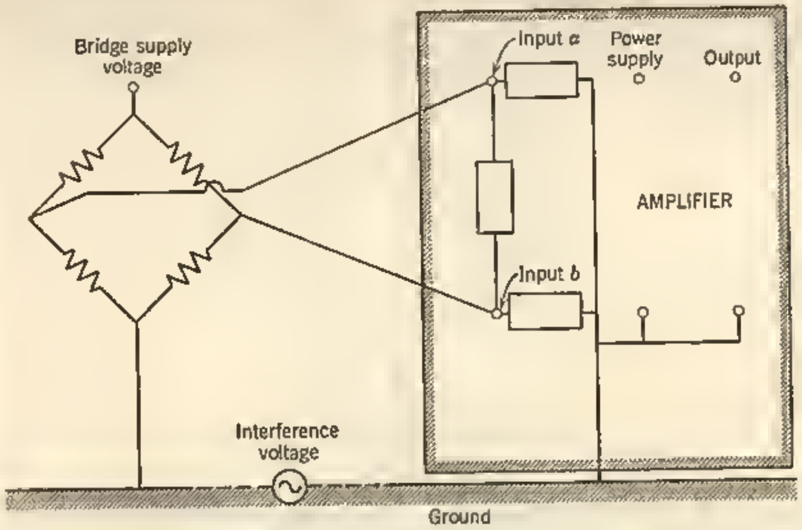


Fig. 3. A "grounded" strain-gauge bridge connected to an amplifier with a floating input. Interference voltages can arise in "ground loops."

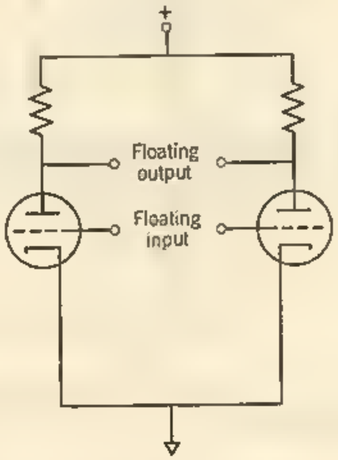


Fig. 4. Circuit for individual amplification of the signal at each terminal of a floating input. The two sides must be exactly balanced.

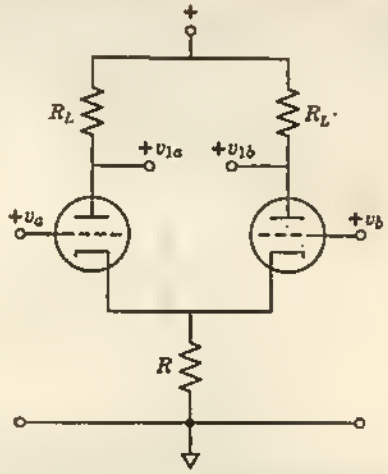


Fig. 5. The basic differential amplifier. The cathode coupling resistance R reduces the common-mode gain without affecting the differential-mode gain.

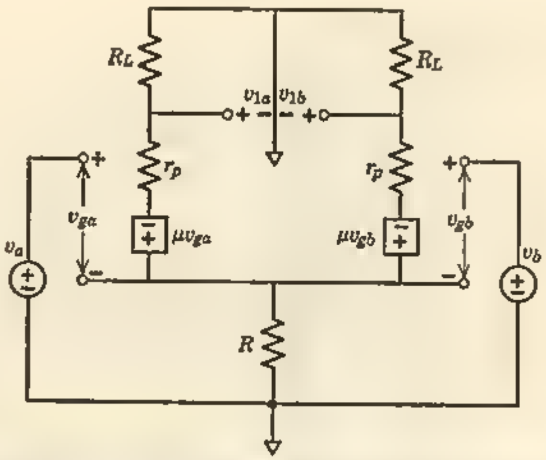


Fig. 6. Equivalent circuit of the basic differential amplifier, for incremental signals. The two sides are assumed identical.

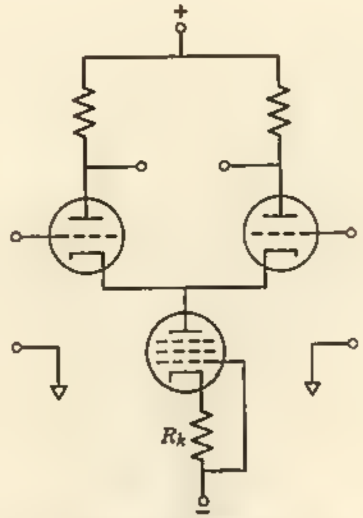


Fig. 7. Replacement of the cathode coupling resistance by a "constant-current" pentode, giving a much improved value of discrimination factor.

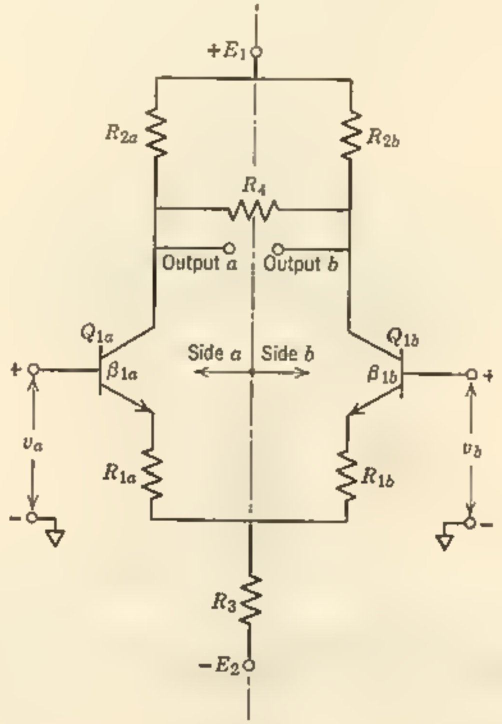


Fig. 8. Transistor d-c differential amplifier, an example of an unbalanced symmetrical circuit.

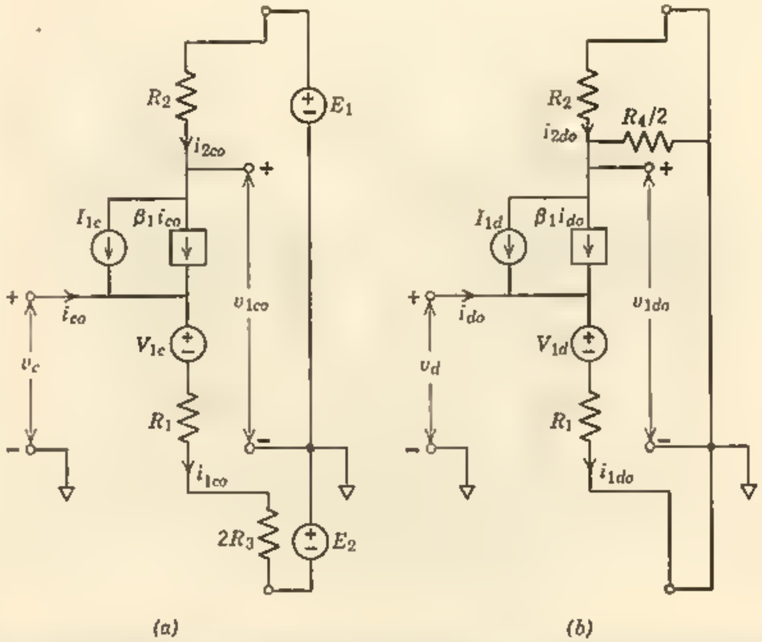
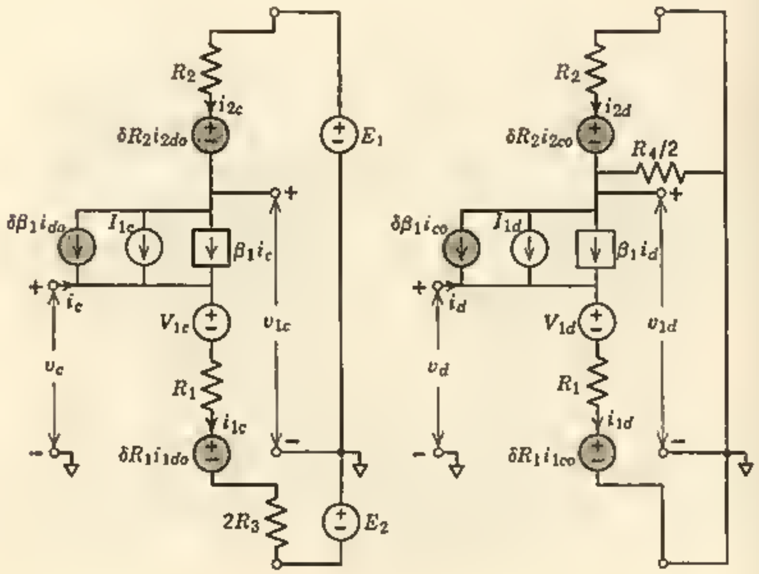


Fig. 9. Common-mode, *a*, and differential-mode, *b*, equivalent half-circuits of Fig. 8, valid when the circuit is balanced. Uncontrolled generators are represented by circles, controlled generators by squares.



(a) (b)

Fig. 10. Common-mode, *a*, and differential-mode, *b*, equivalent half-circuits of Fig. 8, valid when the circuit is unbalanced. The interaction generators are distinguished by shading.

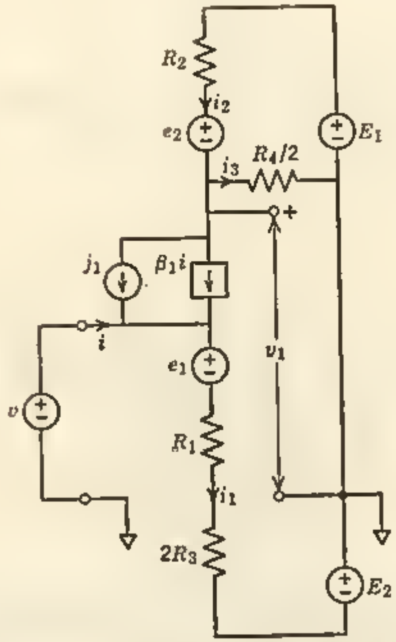


Fig. 11. Composite equivalent half-circuit of Fig. 8, valid when the circuit is unbalanced. Contains the elements of both the common-mode and the differential-mode equivalent half-circuits.

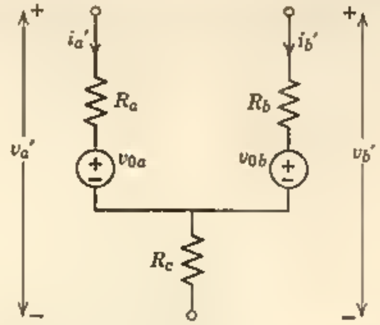


Fig. 12. Equivalent circuit of a typical three-terminal signal source network.

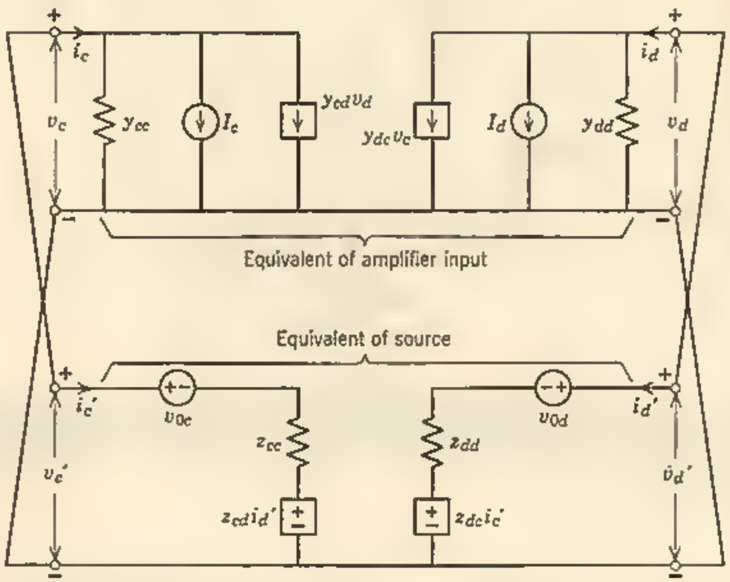


Fig. 13. Representation of signal source by z-parameters, and of differential-amplifier input by y-parameters, in terms of common-mode and differential-mode components of terminal voltages and currents.

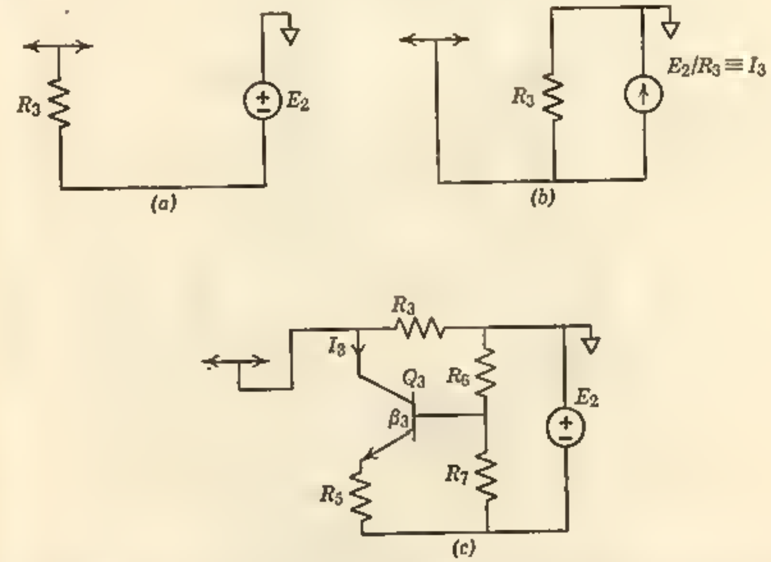


Fig. 14. Stages in the development of an equivalent realization of the emitter circuit in the amplifier of Fig. 8. Part *a* shows the original series resistance and voltage source; part *b*, the conversion to a Norton equivalent; part *c*, the realization of the Norton current generator by a "constant-current" transistor.

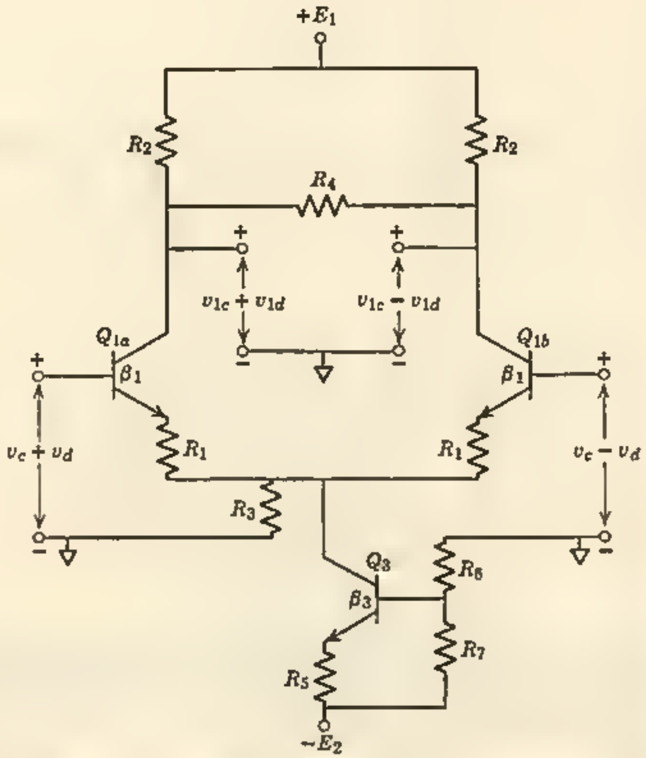


Fig. 15. Transistor d-c differential amplifier with a "constant-current" stage in the emitter circuit.

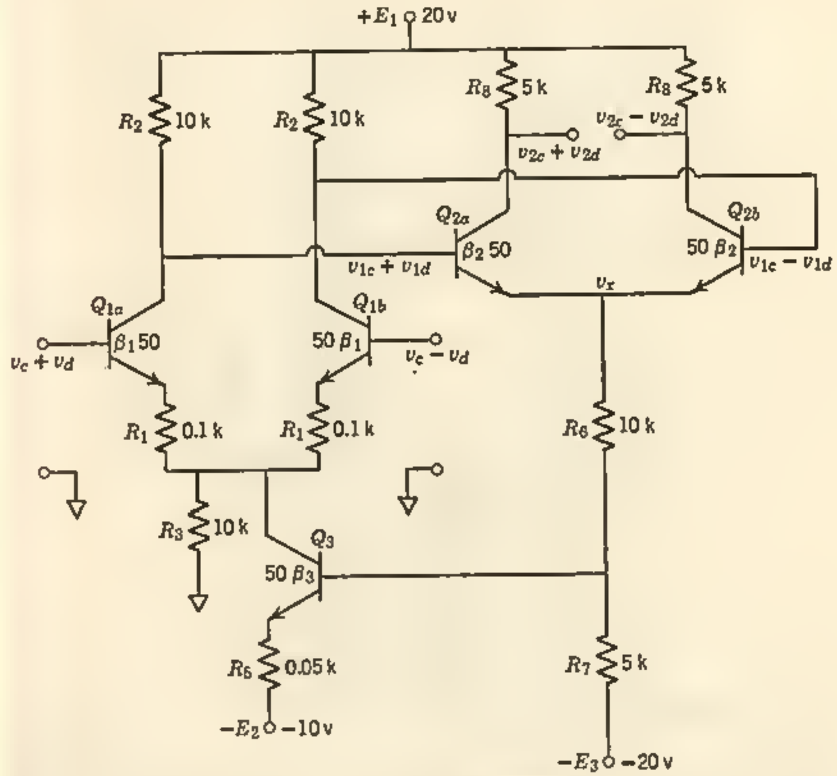


Fig. 16. Two-stage transistor d-c differential amplifier with common-mode negative feedback.

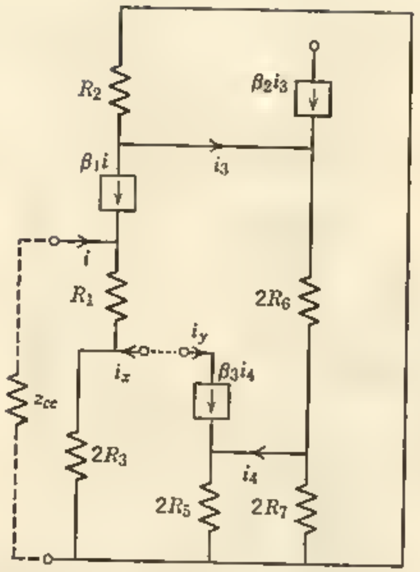


Fig. 17. Common-mode equivalent half-circuit of Fig. 16, showing only the elements essential for calculation of the common-mode loop gain.

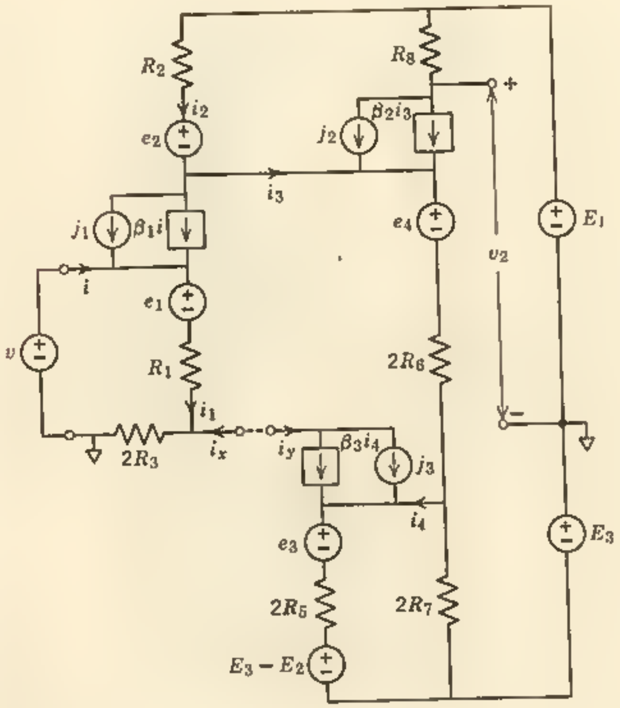


Fig. 18. Composite equivalent half-circuit of Fig. 16, valid when the circuit is unbalanced.

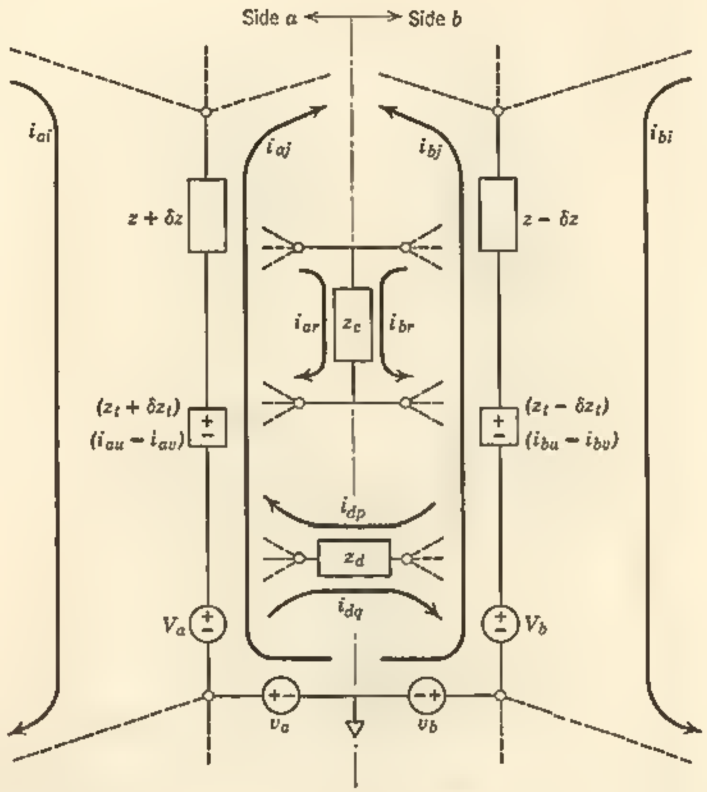


Fig. 19. Generalized form of an unbalanced symmetrical circuit, for analysis on the loop basis.

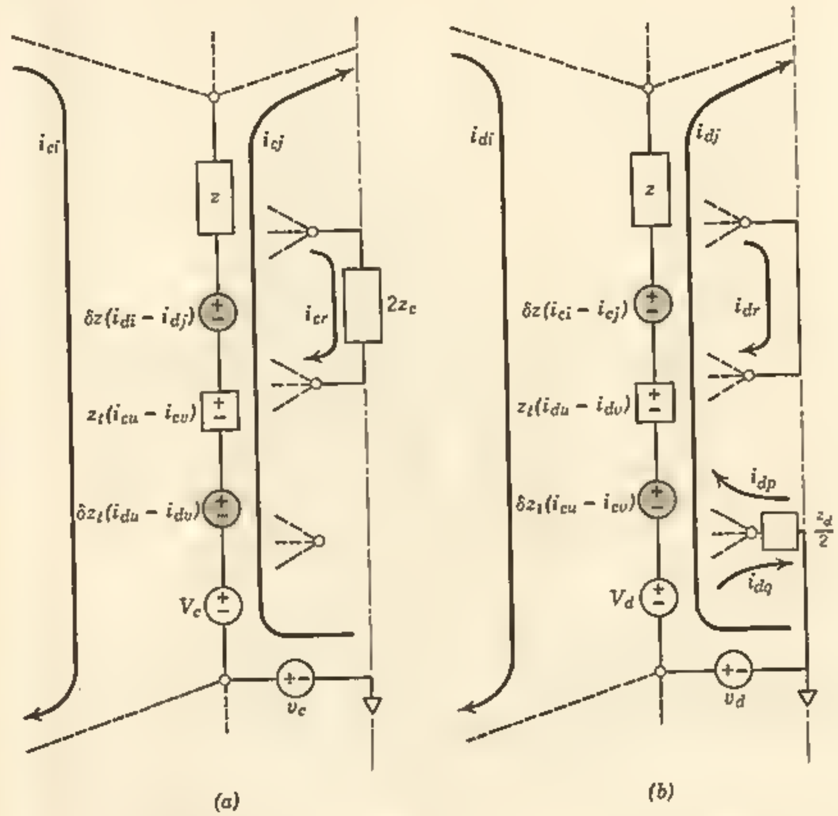


Fig. 20. Common-mode, *a*, and differential-mode, *b*, equivalent half-circuits of Fig. 19, showing interaction generators (shaded).

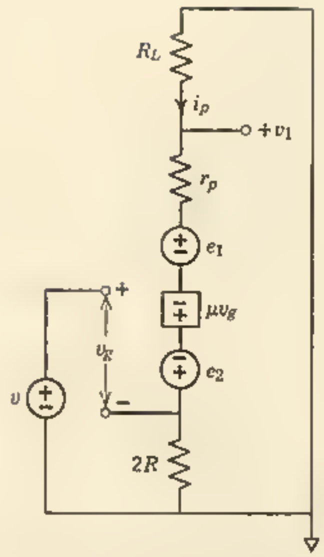


Fig. 21. Composite equivalent half-circuit of Fig. 5. Contains the interaction generators e_1 and e_2, which result respectively from the unbalances in r_p and μ.

Tables

Table I. Definitions of the Important Performance Parameters of a Differential Amplifier

CM and DM gains A_{cc}, A_{dd}, CM-to-DM and DM-to-CM transfer gains A_{dc}, A_{cd}:

$$-A_{cc} \equiv \frac{v_{1c}[v_c]}{v_c} \qquad -A_{cd} \equiv \frac{v_{1c}[v_d]}{v_d}$$

$$-A_{dc} \equiv \frac{v_{1d}[v_c]}{v_c} \qquad -A_{dd} \equiv \frac{v_{1d}[v_d]}{v_d}$$

DM and CM rejection factors H_d, H_c:

$$\frac{1}{H_d} \equiv \frac{v_{1c}[v_d]}{(-A_{cc})v_d} = \frac{A_{cd}}{A_{cc}} \qquad \frac{1}{H_c} \equiv \frac{v_{1d}[v_c]}{(-A_{dd})v_c} = \frac{A_{dc}}{A_{dd}}$$

PS_1 and PS_2 rejection factors H_1, H_2:

$$\frac{1}{H_1} \equiv \frac{v_{1d}[E_1]}{(-A_{dd})E_1} \qquad \frac{1}{H_2} \equiv \frac{v_{1d}[E_2]}{(-A_{dd})E_2}$$

Discrimination factor F and output fractional unbalance U:

$$F \equiv \frac{A_{dd}}{A_{cc}} \qquad U \equiv \frac{v_{1c}[v_d]}{v_{1d}[v_d]} = \frac{A_{cd}}{A_{dd}} = \frac{1}{FH_d}$$

CM and DM equivalent input voltages V_{ci}, V_{di}, produced by internal sources:

$$V_{ci} \equiv \frac{v_{1c}[V_{1c},I_{1c},V_{1d},I_{1d}]}{(-A_{cc})} \qquad V_{di} \equiv \frac{v_{1d}[V_{1d},I_{1d},V_{1c},I_{1c}]}{(-A_{dd})}$$

CM and DM equivalent input voltages V_{ce}, V_{de}, produced by external sources other than the input signals v_c and v_d:

$$V_{ce} \equiv \frac{v_{1c}[E_1,E_2]}{(-A_{cc})} \qquad V_{de} \equiv \frac{v_{1d}[E_1,E_2]}{(-A_{dd})}$$

$$\equiv \frac{1}{A_1}E_1 + \frac{1}{A_2}E_2 \qquad \equiv \frac{1}{H_1}E_1 + \frac{1}{H_2}E_2$$

Total CM and DM equivalent input voltages V_c, V_d, produced by internal sources and external sources other than the input signals v_c and v_d:

$$V_c \equiv V_{ci} + V_{ce} \qquad V_d \equiv V_{di} + V_{de}$$

Table I (Continued)

CM and DM input admittances y_{cc}, y_{dd}, CM-to-DM and DM-to-CM transfer input admittances y_{dc}, y_{cd}:

$$y_{cc} \equiv \frac{i_c[v_c]}{v_c} \qquad y_{cd} \equiv \frac{i_c[v_d]}{v_d}$$

$$y_{dc} \equiv \frac{i_d[v_c]}{v_c} \qquad y_{dd} \equiv \frac{i_d[v_d]}{v_d}$$

CM and DM input currents I_{ci}, I_{di}, produced by internal sources:

$$I_{ci} \equiv i_c[V_{1c},I_{1c},V_{1d},I_{1d}] \qquad I_{di} \equiv i_d[V_{1d},I_{1d},V_{1c},I_{1c}]$$

CM and DM input currents I_{ce}, I_{de}, produced by external sources other than the input signals v_c and v_d:

$$I_{ce} \equiv i_c[E_1,E_2] \qquad I_{de} \equiv i_d[E_1,E_2]$$

Total CM and DM input currents I_c, I_d, produced by internal sources and external sources other than the input signals v_c and v_d:

$$I_c \equiv I_{ci} + I_{ce} \qquad I_d \equiv I_{di} + I_{de}$$

Table II. Example Set of Numbers for the Single-Stage d-c Differential Amplifier of Figure 8

$$R_1 = 0.1\text{ k} \qquad R_2 = 10\text{ k} \qquad R_3 = 10\text{ k}$$

$$\frac{\delta R_1}{R_1} = -0.01 \qquad \frac{\delta R_2}{R_2} = +0.01 \qquad R_4 = 100\text{ k}$$

$$V_{1c} = 0.3\text{ v} \qquad I_{1c} = 0.25\text{ ma}$$

$$V_{1d} = 0.03\text{ v} \qquad I_{1d} = 0.025\text{ ma}$$

$$\frac{\partial V_{1c}}{\partial T} = -0.002\text{ v/°C} \qquad \frac{\partial I_{1c}}{\partial T} = +0.025\text{ ma/°C}$$

$$\frac{\partial V_{1d}}{\partial T} = -0.0002\text{ v/°C} \qquad \frac{\partial I_{1d}}{\partial T} = +0.0025\text{ ma/°C}$$

$$\beta_1 = 50 \qquad E_1 = 20\text{ v}$$

$$\frac{\delta\beta_1}{\beta_1} = 0.1 \qquad E_2 = 20\text{ v}$$

Table III. Example Set of Numbers for the Differential Signal Source of Figure 12

$$R \equiv \frac{R_a + R_b}{2} = 0.5\text{ k} \qquad \delta R \equiv \frac{R_a - R_b}{2} = -0.05\text{ k}$$

$$R_c = 5\text{ k}$$

Table IV. Example Set of Additional Numbers for the Two-Stage d-c Differential Amplifier of Figure 16

$$R_5 = 0.05\text{ k} \qquad R_6 = 10\text{ k}$$

$$R_7 = 5\text{ k} \qquad R_8 = 5\text{ k}$$

$$V_{2c} = 0.3\text{ v} \qquad I_{2c} = 0.25\text{ ma}$$

$$V_{2d} = -0.03\text{ v} \qquad I_{2d} = -0.025\text{ ma}$$

$$\frac{\partial V_{2c}}{\partial T} = -0.002\text{ v/°C} \qquad \frac{\partial I_{2c}}{\partial T} = 0.025\text{ ma/°C}$$

$$\frac{\partial V_{2d}}{\partial T} = 0.0002\text{ v/°C} \qquad \frac{\partial I_{2d}}{\partial T} = -0.0025\text{ ma/°C}$$

$$\beta_2 = 50 \qquad \frac{\delta\beta_2}{\beta_2} = -0.1 \qquad G_c = 14.3$$

$$\beta_3 = 50 \qquad V_{3c} = 0.3\text{ v} \qquad I_{3c} = 0.25\text{ ma}$$

$$E_1 = 20\text{ v} \qquad E_2 = 10\text{ v} \qquad E_3 = 20\text{ v}$$

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